Conversion of Background VTI Depth Model and Full Azimuth Reflection Angle Moveouts into Interval Orthorhombic and/or TTI Layered Parameters

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Summary

We consider a case where full-azimuth reflection angle gatherings were generated using a background VTI depth model. Residual moveouts (RMO) which were automatically picked on these 3D gathers along major horizons indicate considerable periodic azimuthal variations. Our aim is to use the azimuthally dependent RMOs in order to convert the background VTI model into interval TTI and/or orthorhombic layer parameters. Our method is based on a newly derived generalized Dix-based theory, assuming a locally varying 1D TTI, orthorhombic or mixed model, where at each location the vertical orthorhombic axis is the same for all layers but the azimuthal orientations of TTI and orthorhombic layers are different. An effective model for such a layered structure represents a single layer with identical vertical time, effective fast and slow NMO velocities and effective azimuthal orientation of the slow NMO velocity. The NMO velocity and the surface offset azimuth of the effective model coincide with the parameters of the original package of layers for any azimuth of the phase velocity. Our approach starts with a Fourier-based conversion of the RMOs into azimuthally dependent NMO velocities, which are then inverted into three local effective parameters. Finally, we apply a generalized Dix-based inversion approach to estimate the interval orthorhombic or TTI parameters within each layer.

Introduction

Seismic wave propagation along fine horizontal layering is normally characterized by faster velocities in the horizontal direction than in the vertical direction. In the presence of fractures or horizontal tectonic stress, particularly in shale plays, residual moveouts measured along full-azimuth reflection angle gatherings may have a strong azimuthal dependency, indicating faster velocities along an aligned stress/fracture direction. HTI, TTI and orthorhombic model representations take these two properties into account. Parameter estimation of orthorhombic model is analysed in the paper by Bakulin et al. (2000). Inversions of azimuthally dependent NMO velocity in HTI and TTI media have been studied by Tsvankin (1997) and Grechka and Tsvankin (2000). Comprehensive studies on parameter estimation of all practical anisotropic models are given in the book by Tsvankin and Grechka (2011). In this study we use measured (automatically picked) azimuthally dependent RMOs to estimate interval properties of a package of HTI, TTI and orthorhombic layers, where orthorhombic layers share a common vertical axis and all layers may have different azimuthal orientations. For this, we first transform the azimuthally dependent RMOs into NMO velocities, which are then inverted into three effective parameters using a Fourier-based method. Next, we apply a generalized Dix-based inversion to estimate the local fast and slow NMO velocities and the azimuth of the slow NMO velocity at each layer. The solution is unique and independent of the physical nature of the anisotropy in each particular layer. Finally, we use the trend values and apply a constrained minimization to convert the three local NMO parameters (the two velocities and the azimuth) into interval HTI, TTI or orthorhombic parameters at each layer. This work can be considered an extension of our previous study on effective parameters for a package of HTI and VTI layers (Koren et al., 2010). Although we have derived the inversion for both compression and shear waves, in this study we will only deal with compression waves.

Background Model and Input Data

Assume a background depth velocity model consisting of a set of isotropic and VTI layers; each is parameterized with the three interval VTI Thomsen parameters: $V_{ce}$, $\delta$ and $\epsilon$. Using “rich” azimuth seismic data, full-azimuth reflection angle gatherings are generated (Koren and Ravve, 2011) representing background velocity-dependent reflectivities $R(m,\theta,\phi)$ as a function of the phase opening angle $\theta$ and azimuth $\phi$ at given subsurface grid points $m$. Residual moveouts $\Delta t_i(\theta,\phi)$ are then automatically picked along the major horizons $i$, indicating $\pi$-periodic azimuthal variations that can be explained by the existence of aligned stress/fracture systems. Combining the two anisotropic effects of layering (VTI) and horizontally-oriented stress or vertical fractures requires using the orthorhombic model representation. Our goal is to convert the VTI model parameters with the automatically picked RMOs into orthorhombic or TTI interval parameters.

Residual NMO Velocity vs. Residual Moveout

In this section all parameters are considered to be effective, where the background effective VTI NMO velocity is
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related to the effective vertical velocity $V_{\text{ver}}^{\text{VTI}}$ and Thomsen $\delta$ parameter through $V_{\text{ver}}^{\text{VTI}} = V_{\text{ver}}^{\text{ORT}} \sqrt{1 + 2\delta}$. The periodic azimuthally dependent RMOs can be characterized by the fast and slow NMO velocity directions and the relative residual velocities $\phi_{\text{fast}}, \alpha_{\text{fast}}$ (e.g., along fractures) and $\phi_{\text{slow}}, \alpha_{\text{slow}}$ (perpendicular to the fast direction) respectively,

$$\alpha_{\text{fast}} = \frac{V_{\text{ver}}^{\text{ORT}} - V_{\text{nmo}}^{\text{ORT}}}{V_{\text{ver}}^{\text{ORT}}}, \quad \alpha_{\text{slow}} = \frac{V_{\text{ver}}^{\text{ORT}} - V_{\text{nmo}}^{\text{ORT}}}{V_{\text{ver}}^{\text{ORT}}}$$  \hspace{1cm} (1)

where the fast and slow local NMO velocities are related to the interval parameters. In case of an orthorhombic effective model, these are the vertical velocity of the orthorhombic medium $V_{\text{ver}}^{\text{ORT}}$ and the orthorhombic Thomsen parameters,

$$V_{\text{nmo}}^{\text{fast}} = V_{\text{ver}}^{\text{ORT}} \sqrt{1 + 2\delta_{1}}, \quad V_{\text{nmo}}^{\text{slow}} = V_{\text{ver}}^{\text{ORT}} \sqrt{1 + 2\delta_{2}}.$$  \hspace{1cm} (2)

We choose the horizontal axes in such a way that $\delta_{1} > \delta_{2}$. Note that in the case where $\delta_{1} = \delta_{2}$ (or $\alpha_{\text{fast}} = \alpha_{\text{slow}}$) there is no azimuthal dependency (like in VTI media).

Equations 1 and 2 lead to

$$\delta_{1} = \frac{(1 + \alpha_{\text{fast}}) V_{\text{ver}}^{\text{ORT}}^{2} - V_{\text{ver}}^{\text{ORT}}}{2 V_{\text{ver}}^{\text{ORT}}^{2}}, \quad \delta_{2} = \frac{(1 + \alpha_{\text{slow}}) V_{\text{ver}}^{\text{ORT}}^{2} - V_{\text{ver}}^{\text{ORT}}}{2 V_{\text{ver}}^{\text{ORT}}^{2}}.$$  \hspace{1cm} (3)

In case of a TTI effective model, the fast NMO velocity may correspond to the dip direction, and the slow NMO velocity to the strike direction, or vice versa. The dip and strike NMO velocities depend on the vertical phase velocity

$$V_{\text{phs}}(\theta_{\text{ax}})$$

and its derivatives, which in turn, depend on the axial compression velocity $V_{p}$ and Thomsen TI parameters $\phi$ and $\epsilon$, and on the tilt angle $\delta_{\epsilon}$ of the TTI.

$$V_{\text{nmo}}^{\text{dip}} = \sqrt{V_{\text{phs}}^{2}(\theta_{\text{ax}})} + V_{\text{phs}}(\theta_{\text{ax}}) V_{p}^{\epsilon}(\theta_{\text{ax}}).$$  \hspace{1cm} (4)

We have derived a general relation for NMO velocity vs. the azimuth of the phase velocity $\phi$ or $\Delta \phi = \phi - \phi_{\text{slow}}$.

$$V_{\text{nmo}}^{2}(\Delta \phi) = \frac{V_{\text{slow}}^{2} \cos^{2} \Delta \phi + V_{\text{fast}}^{2} \sin^{2} \Delta \phi}{V_{\text{slow}}^{2} \cos^{2} \Delta \phi + V_{\text{fast}}^{2} \sin^{2} \Delta \phi}.$$  \hspace{1cm} (5)

Introduction of 1 into 5 also leads to

$$V_{\text{nmo}}^{2}(\Delta \phi)/V_{\text{ver}}^{\text{ORT}}^{2} = \frac{(1 + \alpha_{\text{slow}}) \cos^{2} \Delta \phi + (1 + \alpha_{\text{fast}}) \sin^{2} \Delta \phi}{(1 + \alpha_{\text{slow}}) \cos^{2} \Delta \phi + (1 + \alpha_{\text{fast}}) \sin^{2} \Delta \phi} = f(\Delta \phi).$$  \hspace{1cm} (6)

The relative residual NMO velocity can be linearized for relatively small moveouts and weak anisotropy,

$$\Delta V_{\text{nmo}}^{\phi} = \frac{V_{\text{ver}}^{\text{ORT}} - V_{\text{nmo}}^{\text{ORT}}}{V_{\text{ver}}^{\text{ORT}}} \approx \alpha_{\text{fast}} \sin^{2} \Delta \phi + \alpha_{\text{slow}} \cos^{2} \Delta \phi.$$  \hspace{1cm} (7)

The RMO can be then written as

$$\Delta f(\Delta \phi, \theta) = -{(1 + 2\delta)} \sin^{2} \theta \Delta V_{\text{nmo}}^{\phi}(\Delta \phi) = -{(1 + 2\delta)} \sin^{2} \theta \left[\alpha_{\text{fast}} \sin^{2} \Delta \phi + \alpha_{\text{slow}} \cos^{2} \Delta \phi\right].$$  \hspace{1cm} (8)

For weak anisotropy, Grechka and Tsankin (1998) derived a similar form of relation for moveouts measured along surface offset time gathers representing variations with respect to the ray velocity azimuth. In our case the derivation is done for residual moveouts directly in the phase angle domain.

Effective Model Parameters from Azimuthally Dependent RMO

In this section we obtain three effective RMO azimuthal parameters $\alpha_{\text{fast}}, \alpha_{\text{slow}}, \phi_{\text{slow}}$ from the automatically picked RMOs at each reflecting point. Generally, these kind of problems can be solved by, for example, a least square fit approach; in our case, however, we use the natural periodic form of the input RMO data to analytically establish the three effective parameters. We first use equation 8 to transform the RMOs into azimuthally-dependent NMO velocities $V_{\text{nmo}}(\phi)$. We then use equation 5, which is a periodic function of the phase velocity azimuth with period $\pi$. Assume that the data is sampled regularly on the interval $[0, \pi]$ starting from an arbitrary reference azimuth. We expand equation 6 into the Fourier series,

$$f(\phi) = a_{c} + \sum_{k=1}^{\infty} a_{k} \cos(2k\phi) + \sum_{k=1}^{\infty} b_{k} \sin(2k\phi),$$  \hspace{1cm} (9)

where the coefficients of the expansion are delivered by,

$$a_{c} = \frac{1}{\pi} \int_{0}^{\pi} f(\phi) \cos(2k\phi) d\phi,$$$$

b_{k} = \frac{2}{\pi} \int_{0}^{\pi} f(\phi) \sin(2k\phi) d\phi.$$  \hspace{1cm} (10)

Introduction of equation 6 into 10 yields the AC and the DC terms,

$$a_{c} = (1 + \alpha_{\text{fast}}) \frac{1}{I + \alpha_{\text{slow}} + (\alpha_{\text{fast}} - \alpha_{\text{slow}})^{2}},$$

$$a_{k} = m_{k} \cos(2k\phi_{\text{ax}}), \quad b_{k} = m_{k} \sin(2k\phi_{\text{ax}})$$  \hspace{1cm} (11)

where the magnitude of the DC terms reads,
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\[ m_k = -2(1 + \alpha_{\text{fast}})(1 + \alpha_{\text{slow}}) \left( \frac{\alpha_{\text{fast}} - \alpha_{\text{slow}}}{2 + \alpha_{\text{fast}} + \alpha_{\text{slow}}} \right)^k \]

(12)

Coefficients \( a_k \) and \( b_k \) can be presented as real and imaginary parts of a complex number. The slow velocity azimuth can be uniquely obtained from the first pair, \( 2\phi_{\text{slow}} = \arctan(b_1/a_1) + \pi \). The second pair gives two solutions, and one of these solutions (the proper solution) should coincide with the axis azimuth obtained previously. An arbitrary pair \( l \) gives \( l \) solutions. Only one of these solutions is correct, and for NMO velocity values characterized by reflections from azimuthally-dependent layered models, it should coincide with the unique solution for \( \phi_{\text{slow}} \). To get the Fourier coefficients, we apply the discrete forward Fourier transform to the residual NMO velocity \( \Delta V_{\text{nmo}}(\varphi) \) defined as

\[ \Delta V_{\text{nmo}} = V_{\text{nmo}} - V_{\text{ef}} \]

on the interval \( 0 \leq \varphi < \pi \), and we get the normalized AC and DC coefficients \( F_0 \) and \( F_k \). We obtain parameters \( \alpha_{\text{slow}} \) and \( \alpha_{\text{fast}} \) defined above,

\[ \alpha_{\text{slow}} = F_0 - \sum_{k=1}^{N} |F_k|, \quad \alpha_{\text{fast}} = F_0 - \sum_{k=1}^{N} (-1)^k |F_k| \]

(13)

where \( N \) is the number complex terms in the Fourier space. The absolute values \( |F_k| \) decay very quickly, and five terms normally suffice. The effective parameters are \( \alpha_{\text{slow}}, \alpha_{\text{fast}} \) and \( \phi_{\text{slow}} \).

Inversion from Effective NMO Velocities to Local NMO Layer Velocities

In this section we find the fast and slow local NMO velocities and the azimuth of the slow NMO velocity for HTI, TTI and orthorhombic layers, using the already known effective parameters at the top and bottom horizons of each layer. We define auxiliary effective factors \( W_{x,n} \), \( W_{y,n} \) and \( U_n \) for each horizon \( n \) as,

\[ W_{y,n} = \sum_{i=1}^{n} V_{\text{nmo},i}^{\text{fast},2} - V_{\text{nmo},i}^{\text{slow},2} \left( \frac{\alpha_{\text{fast}} - \alpha_{\text{slow}}}{2 + \alpha_{\text{fast}} + \alpha_{\text{slow}}} \right)^2 \sin 2\phi_{\text{slow},i} \Delta t_{O,i,i} \]

\[ W_{x,n} = \sum_{i=1}^{n} V_{\text{nmo},i}^{\text{fast},2} - V_{\text{nmo},i}^{\text{slow},2} \left( \frac{\alpha_{\text{fast}} - \alpha_{\text{slow}}}{2 + \alpha_{\text{fast}} + \alpha_{\text{slow}}} \right)^2 \cos 2\phi_{\text{slow},i} \Delta t_{O,i,i} \]

\[ U_n = \sum_{i=1}^{n} V_{\text{nmo},i}^{\text{fast},2} + V_{\text{nmo},i}^{\text{slow},2} \left( \frac{\alpha_{\text{fast}} - \alpha_{\text{slow}}}{2 + \alpha_{\text{fast}} + \alpha_{\text{slow}}} \right)^2 \Delta t_{O,i,i} \]

(14)

where \( \Delta t_{O,i,i} \) is the two-way vertical time for a layer, and the summation is performed over the \( n \) layers with the unknown (so far) local parameters. These factors can be directly computed from the known effective parameters,

\[ W_{x,n} = \frac{V_{\text{nmo},i}^{\text{fast},2} - V_{\text{nmo},i}^{\text{slow},2}}{2} \cos 2\phi_{\text{slow},i} t_{O,n,i} \]

\[ W_{y,n} = \frac{V_{\text{nmo},i}^{\text{fast},2} - V_{\text{nmo},i}^{\text{slow},2}}{2} \sin 2\phi_{\text{slow},i} t_{O,n,i} \]

\[ U_n = \frac{V_{\text{nmo},i}^{\text{fast},2} + V_{\text{nmo},i}^{\text{slow},2}}{2} t_{O,n,i} \]

(15)

The interval parameters then follow from definition 14,

\[ V_{\text{nmo},n}^{\text{fast},2} = \frac{\Delta U_n + \Delta W_n}{\Delta \varphi} \] \[ V_{\text{nmo},n}^{\text{slow},2} = \frac{\Delta U_n - \Delta W_n}{\Delta \varphi} \]

\[ 2\phi_{\text{slow},n} = \arctan \frac{\Delta W_{n}}{\Delta U_{n}} \]

(16)

where the following notations are used,

\[ \Delta W_{n} = W_{x,n} - W_{x,n-1} \]

\[ \Delta W_{n} = W_{y,n} - W_{y,n-1} \]

\[ \Delta U_{n} = U_{n} - U_{n-1} \]

Thus, we obtain the fast and the slow local NMO velocities and the azimuth of the slow NMO velocity.

Inversion from Local NMO Velocities to Interval Parameters of Orthorhombic or TTI Layers

For each individual layer, two physical features have been obtained: the fast and the slow NMO velocity. The simplest azimuthally dependent model is HTI, it can be described by two interval parameters only: the vertical compression velocity \( V_{\text{ver},i} \) and Thomsen parameter \( \delta_{2,i} \). In this case the solution is unique (Koren et al., 2010). However, for an orthorhombic layer, there are three parameters, \( V_{\text{ver},i}, \delta_{2,i} \) and \( \epsilon_{i} \), while for a TTI layer there are four interval parameters: the axial velocity \( V_{\text{ax},i} \), Thomsen parameters \( \delta_{1,i} \) and \( \delta_{2,i} \), and the tilt of the axis of symmetry \( \theta_{\text{ax},i} \). To get the solution, we assume that the interval parameters should be as close as possible to the given trend, and we consider the given values of the fast and the slow NMO velocities as the constraints. In case of the orthorhombic model, the target function reads,

\[ \alpha_{n}^2 + (\delta_{2,i} - \delta_{n})^2 + (\delta_{2,i} - \delta_{n})^2 \rightarrow \min \]

where \( \alpha_{n} = |V_{\text{ver},n} - V_{\text{TTI},n}|/V_{\text{ver},n} \), and \( \delta_{n} \) belongs to the background VTI medium. The constraints for the orthorhombic model are given in equation 2. The Thomsen parameters \( \delta_{1,i} \) and \( \delta_{2,i} \) can be eliminated from the constraints, and we obtain the target
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function with the vertical velocity only. In case of the TTI model, the constraints are listed in equation 4, where the magnitude of the phase velocity related is to the angle between its direction and the TI axis (Tsvankin, 2001),

\[ \frac{V_{\text{phs}}^2}{V_P^2} = 1 - f \left( \frac{\pi}{2} + 2 \sin^2 \alpha_{\text{phs}} + \frac{1}{2} \sqrt{\left( f + 2 \sin^2 \theta_{\text{ax}} \right)^2 - 2 f (\varepsilon - \delta) \sin^2 2 \theta_{\text{ax}}} \right) \].

For a TTI layer, we solve a constrained minimization problem applying the Lagrangian multipliers method.

Example

Figure 1 displays a depth image of a 3D model from the Barnett Shale. A VTI background velocity was used to create the image and to produce 3D high-resolution full-azimuth reflection angle gathers. Figure 2 shows an example of this kind of gather, where only azimuthal varying traces with a given opening angle of 60 degrees are displayed. Automatic RMO picking was performed along the top and bottom horizons of the Barnett formation (Marble Falls and Ellenburger, respectively). Figure 3 displays a zoom-in of the Ellenburger reflection event, where periodic RMOs can be clearly seen. We first computed the three effective RMO parameters for each reflection event. We then used the constrained generalized Dix method to convert the effective parameters into four interval orthorhombic parameters. The effective data and the interval parameters are listed in Tables 1 and 2.

Table 1: Effective data

<table>
<thead>
<tr>
<th></th>
<th>Horizon 1</th>
<th>Horizon 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Back RMS vel. [m/s]</td>
<td>3569</td>
<td>3716</td>
</tr>
<tr>
<td>Back. vertical vel. [m/s]</td>
<td>3500</td>
<td>3622</td>
</tr>
<tr>
<td>( \alpha_{\text{fast}} )</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>( \alpha_{\text{slow}} )</td>
<td>-0.05</td>
<td>-0.065</td>
</tr>
<tr>
<td>( \varphi_{\text{slow}} )</td>
<td>70°</td>
<td>92°</td>
</tr>
<tr>
<td>Two-way vertical time [ms]</td>
<td>750</td>
<td>900</td>
</tr>
</tbody>
</table>

Table 2: Inverted interval parameters

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \delta_1 )</th>
<th>( \delta_2 )</th>
<th>( \varphi_{\text{ax}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.063</td>
<td>0.19</td>
<td>-0.23</td>
<td>112°</td>
</tr>
</tbody>
</table>

Conclusions

In this work we describe a novel method for updating a background VTI depth layered model into an HTI, TTI or orthorhombic layered model with common vertical axis for orthorhombic layers and different azimuthal orientations of the horizontal orthorhombic axes or TI axes of symmetry. A key component in our approach is the ability to obtain high-resolution residual moveouts which are automatically picked along 3D full-azimuth reflection angle gathers at the major horizons. The residual moveouts are converted to azimuthally-dependent NMO velocities, which are then inverted to three effective parameters. A generalized constrained Dix-based inversion is then used to convert the effective parameters along the horizons first into three local parameters for each layer: the fast and the slow local NMO velocities and the azimuth of the slow NMO velocity. The two NMO velocities are further converted into interval parameters for each layer: two parameters for the HTI (\( V_{\text{ver}} \) and \( \delta_2 \)), three parameters for the orthorhombic model (\( V_{\text{ver}}, \delta_1, \delta_2 \)), and four parameters for the tilted TI (axial velocity \( V_P, \delta, \varepsilon \), and the axis tilt \( \theta_{\text{ax}} \)). In all cases except the HTI, we introduce the trend values of the parameters to get a well-defined constrained minimization problem with a unique solution.