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Eigenray Tracing in 3D Heterogeneous Media

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Summary

Conventional two-point ray tracing in a general 3D heterogeneous medium is normally performed by a shooting method. The location and the slowness components are specified at the start point, and the ray path is the solution of ODEs with the initial conditions. The ray arrives to some proximity of the destination, and the start direction is then successively refined, so that the ray path eventually includes the destination point. Eigenray tracing, however, is a boundary-value problem, rather than an initial-value problem. The boundary conditions are two endpoint locations, and the ray trajectory satisfies Fermat's principle of least traveltime. In this study, we apply the non-linear Finite Element Analysis to find the least-time ray path. The ray trajectory is split into a number of three-nodal segments with quadratic interpolation of trajectory points, traveltime and other functions between the nodes. For each segment, we compute the traveltime, and its first and second derivatives with respect to the nodal locations. The local derivatives (related to a single segment) are combined into the global derivatives of the entire path. For the least time, the first derivatives vanish. Knowledge of the second derivatives makes it possible to apply the Newton method for ray path optimization.

Introduction

Conventional two-point ray tracing in general 3D heterogeneous media is normally performed using the so-called shooting method. In this method, a fan of rays is traced from a given starting point, where the rays arriving in the vicinity of the target location, with close take-off angles, are used to converge to the actual end point. By covering a wide range of take-off angles, multi-pathing stationary solutions can be found. However, in complex geological areas, the solution to the initial-value problem associated with the individual rays is highly sensitive to small changes in the take-off angles at the starting point, resulting in shadow zones which the numerical rays cannot penetrate. The proposed Eigenray method (also known as the ray bending method) is a direct solution of the boundary-value problem, rather than the initial-value problem. The boundary conditions are the two endpoint locations, and the ray trajectory satisfies Fermat's principle of stationary traveltime. This method is particularly attractive for areas that involve sharp velocity variations or local velocity anomalies. The case of simulating head waves is an extreme example of Eigenrays providing stable solutions where conventional ray tracing fails. Ray bending methods have been extensively studied. Westwood and Vidmar (1987) applied the method to simulate the signals interacting with a layered ocean bottom. Waltham (1988) studied models consisting of constant-velocity layers separated by curved interfaces and computed ray paths whose traveltimes are stationary with respect to changes in ray/interface intersection positions. The amplitude of the resulting event is related to the second derivative of the traveltime with respect to changes in the position of the ray/interface intersection. Moser (1991) used this method to compute the traveltime between the source point and all points of a network. Moser et al. (1992) improved the conventional ray bending approach by applying a) gradient search methods and b) interpolation by beta-splines between the nodes. Farra (1992) applied the Hamiltonian formulation to the ray bending approach. Shashidhar and Anand (1995) solved the problem of three-dimensional Eigenray tracing in an ocean channel. Bona et al. (2009) demonstrated that Fermat's principle of stationary traveltime holds for general heterogeneous anisotropic media using stochastic simulated annealing global search methods. Recently, Sripanich and Fomel (2014) presented an efficient algorithm for two-point seismic ray tracing in layered media by means of the bending method, where the ray paths are discretized at the intersection of the rays with the structure's interfaces. In this study, we demonstrate the power of applying this type of ray tracing by using a non-linear spectral element method to efficiently find accurate least-time ray paths in complex geological areas, providing solutions beyond conventional ray tracing.

Model and Method

Consider an initial ray trajectory that is divided into a number of multi-nodal segments (spectral finite elements) with a Legendre polynomial interpolation. Each segment includes two endpoints and a number of internal points placed along the internal flow parameter according to the Gauss-Lobatto quadrature. The quadrature also associates definite weights for all points of a segment. The simplest spectral element is a three-nodal segment which we use to demonstrate the synthetic examples. The nodes are located at -1, 0 and $+1$ of the flow parameter. For each segment, we compute the traveltime and its first and second derivatives with respect to the nodal locations. The local derivatives (related to a single segment) are combined into the global derivatives of the entire path. The local derivatives are vectors of size $3n$, and the local second derivatives are matrices of the same size, where n is the number of segment nodes, including its endpoints. The global matrix of the second derivatives has a band structure with a narrow band width $3n$. This matrix is symmetric, and if the stationary traveltime is minimum (the most practical case), it is also positive-definite in the proximity of the stationary ray. For the least-time solution, the first derivatives vanish. Knowledge of the second derivatives makes it possible to apply the Newton method for Eigenray optimization. The second derivatives of the traveltime depend on the locations of the nodes, and thus the minimization problem is nonlinear. In addition, the second derivatives may be used to compute the geometrical spreading when the stationary ray path is found. We note that the least traveltime (or generally, the stationary traveltime) fully defines the ray path, but it still allows moving the nodes along the ray. Therefore, we apply an additional constraint on the lengths between the successive nodes. The nodes are located more densely in regions where the velocity changes rapidly. Thus, the constraints are the ratios between the arc lengths connecting successive nodes. There are, in general, two ways to implement the constraints: stiff constraints (e.g., applying Lagrangian multipliers' method) and soft or relaxed constraints, adding a penalty term to the target function to be minimized. The soft constraint method is simpler, does not lead to additional unknown parameters, does not increase the bandwidth of the resolving matrix, and still provides excellent accuracy. Since the constraints only affect the location of the nodes along the stationary ray, the computational formulae are essentially simplified if the chords (i.e., the distances between the successive nodes) are applied instead of the arc lengths.

Synthetic Examples

Three examples of ray bending solutions are presented.

Example 1: High-velocity half-space under constant velocity layer (head wave). Consider a 1D velocity model whose profile and gradient are shown in Figure 1, to the left.

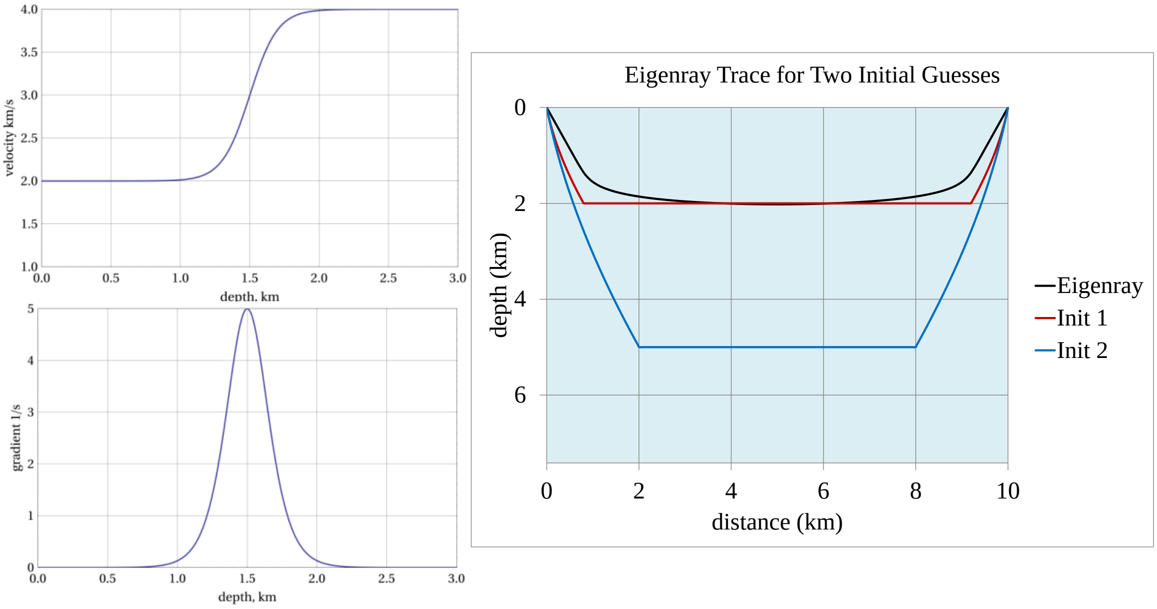

Figure 1 *High-velocity half-space under constant velocity layer: Velocity model and stationary ray*

The smoothed velocity model is depth-dependent and described by,

$$v = v_0 + \frac{\Delta v}{2}\left(1 + \tanh\frac{z - z_t}{z_0}\right) , \tag{1}$$

where $v_0 = 2\,\text{km/s}$ is the velocity of the "homogeneous" layer above the half-space, $\Delta v = 2\,\text{km/s}$ is the difference between the velocity of the half-space and that of the overlying layer; thus, the half-space velocity is $v_h = v_0 + \Delta v = 4\,\text{km/s}$. Actually, neither the overlying layer nor the half-space is homogeneous due to the transition zone. Parameter $z_t = 1.5\,\text{km}$ is the mid-level of the transition zone, and parameter $z_0 = 0.2\,\text{km}$ is the characteristic distance that shows the width of the transition zone. The offset $h = 10$ km. Figure 1, to the right, shows the initial guesses and the stationary ray. We applied the initial guess twice: first with the floor depth 2 km (which is approximately the lower end of the transition zone), then with the over-estimated floor depth 5 km (red and blue lines, respectively). Both initial guesses converge to a unique head wave solution (black line). In the next example we show that in the case of multi-arrivals, the final solution is sensitive to the initial guess.

Example 2: One-way path in a medium with low-velocity elliptic anomaly. Consider a constant background velocity with an elliptic anomaly region of a lower velocity, as shown in Figure 2. Location of the ellipse center is x_c, z_c, and the semi-axes of the ellipse are r_x, r_z. The velocity field is described by an analytic function,

$$v(x,z)=v_b+\frac{\Delta v}{2}(\tanh A-1) \quad , \qquad A=\frac{1}{s}\left[\frac{(x-x_c)^2}{r_x^2}+\frac{(z-z_c)^2}{r_z^2}-1\right] \quad , \tag{2}$$

where v_b is the background velocity outside the ellipse, and $v_b-\Delta v$ is the anomalous low velocity inside the ellipse. Negative Δv leads to anomalous high velocity inside the ellipse. Parameter s is the smoothing scale: the smaller s is, the sharper the velocity change. For infinitesimal s, the velocity function becomes discontinuous. We accept the following parameters,

$$v_b=5\text{km/s},\ \Delta v=3\text{km/s},\ x_c=5\text{km},\ z_c=3\text{km},\ r_x=3\text{km},\ r_z=2\text{km},\ s=0.2 \quad . \tag{3}$$

The source is located at the subsurface point with zero horizontal coordinate and depth $d_z=6\text{km}$, and the receiver is on the surface, with the one-way offset $d_s=10\text{km}$. Three different initial guesses shown in Figure 2 left, by green, black and blue lines, lead to three different solutions shown in Figure 2 right. The red ellipse is the contour of the velocity anomaly.

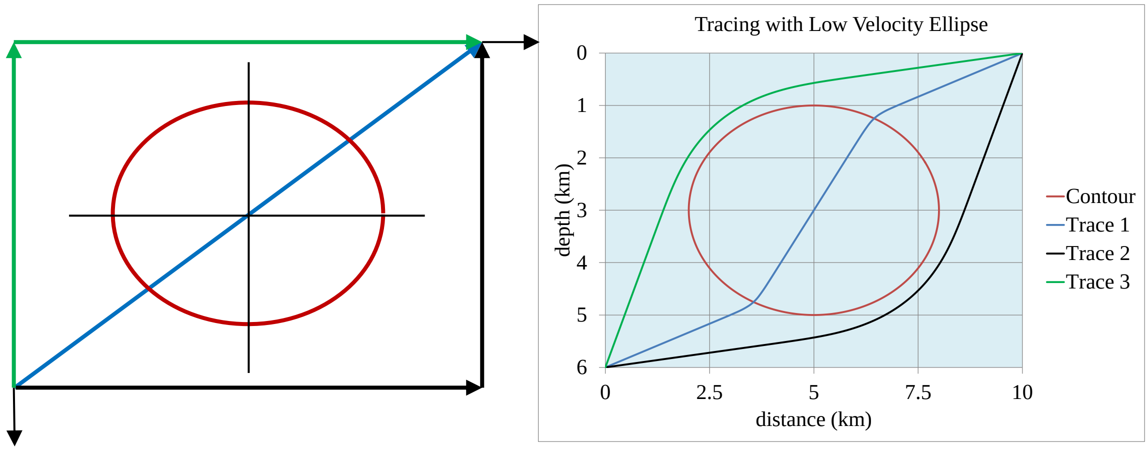

Figure 2 *One-way path in a medium with low-velocity elliptic anomaly*

For rays 1 and 2 (black and green lines), the resulting (minimum) traveltime is $t=2.61048\text{s}$, and for ray 3 (blue line), the traveltime is $t=3.71291\text{s}$. Rays 1 and 2 are symmetric solutions, where the ray bypasses the low-velocity inclusion and almost avoids penetration into the transition zone. These rays travel completely through the high-velocity background. For ray 3, refraction occurs twice, upon entry to and exit from the low-velocity ellipse. Thus, we deal with a multi-arrival case, characterized by three local minima, two of which are also global.

<u>Example 3: Velocity field with two elliptic anomalies.</u> Consider a model that combines slow- and high-velocity anomalies and fast half-space. It can be analytically described by,

$$v(x,z)=v_b-\frac{\Delta v}{2}(1-\tanh A_1)+\frac{\Delta v}{2}(1-\tanh A_2)+\frac{\Delta v_h}{2}(1+\tanh A_3) \quad , \tag{4}$$

where

$$A_1\equiv\frac{1}{s}\left[\frac{(x-x_1)^2}{r_x^2}+\frac{(z-z_c)^2}{r_z^2}-1\right], \quad A_2\equiv\frac{1}{s}\left[\frac{(x-x_2)^2}{r_x^2}+\frac{(z-z_c)^2}{r_z^2}-1\right], \quad A_3\equiv\frac{z-z_o}{z_d} \quad . \tag{5}$$

Parameters x_1 and x_2 are horizontal coordinates of central points of elliptic anomalies, z_c is their common vertical coordinate, z_o is the floor depth of the high-velocity half-space. Parameters s and z_d govern the width of the transition zones,

$$\begin{gathered} v_b=4\text{km/s},\ \Delta v=2\text{km/s},\ \Delta v_h=3\text{km/s},\ x_1=4\text{ km},\ x_2=20\text{ km}, \\ z_c=2\text{ km},\ r_x=4\text{km},\ r_z=1\text{km},\ z_o=4\text{km},\ z_d=0.2\text{km},\ s=0.2, \end{gathered} \tag{6}$$

and the offset $h = 22$ km. Figure 3 shows the velocity field (left) and the ray tracing results (right). The distances in the velocity field plot are in km, and velocity is in km/s.

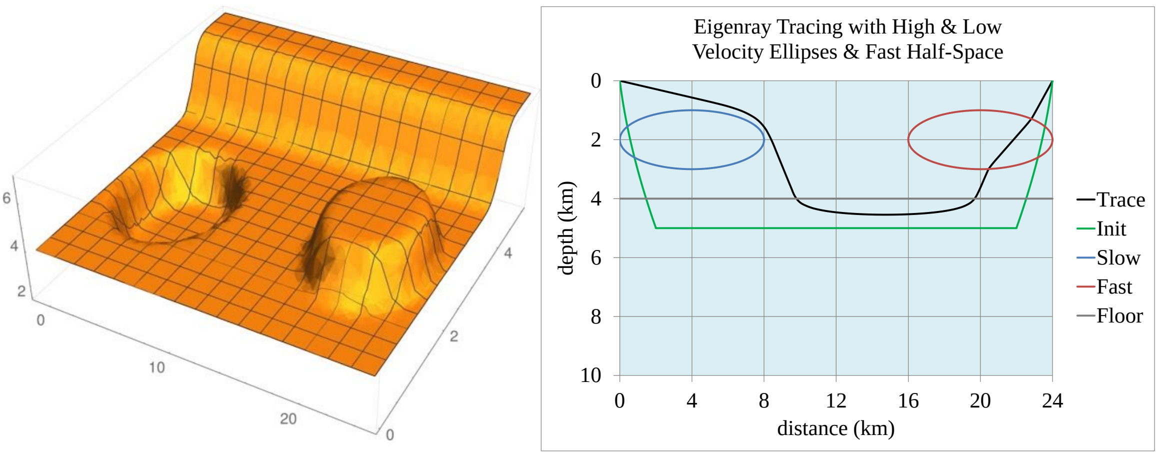

Figure 3 *Ray tracing in a medium with two elliptic anomalies and high-velocity half-space*

In the right subplot, the blue and red ellipses show the low-velocity and high-velocity anomalies, respectively. The gray line is the location of the half-space floor midline. The green line is the initial guess, where the depth of the floor was deliberately overestimated, in order to demonstrate convergence to a correct result even with an initial error. The black line is the least-time ray path.

Conclusions

A ray bending algorithm, referred to as the Eigenray method, has been developed for two-point tracing in a general 3D heterogeneous medium. The method uses spectral elements with Legendre polynomial interpolation between the nodes and Gauss-Lobatto quadrature. Additional soft constraints related to the segment lengths govern the locations of the nodes along the stationary ray. Explicit expressions for the traveltime and its first and second derivatives allow the implementation of the Newton method optimization, which also yields the geometrical spreading. We demonstrated the attractiveness of the method in cases of local low/high velocity anomalies and for solving the ray trajectory of head waves. For multi-arrivals, the final solution is obviously sensitive to the initial guess. A strategy of combining the global ray shooting method with the proposed Eigenray method seems very attractive.

References

Bona, A., Slawinski, M., and Smith, P. [2009]. Ray tracing by simulated annealing: Bending method. Geophysics, 74, no. 2, T25–T32.
Farra, V. [1992]. Bending method revisited: a Hamiltonian approach. Geophysical Journal International, 109, no. 1, 138-150.
Moser, T. [1991]. Shortest path calculation of seismic rays, Geophysics, 56, no. 1, 59-67.
Moser, T., Nolet, G., and Snieder, R. [1992]. Ray bending revisited, Bulletin of the Seismological Society of America, 82, no. 1, 259-288.
Shashidhar, N., and Anand, G. [1995]. Eigenray tracing in an ocean using Fermat's principle. Journal of Sound and Vibration, 186, no. 2, 231-243.
Sripanich, Y., and Fomel, S. [2014]. Two-point seismic ray tracing in layered media using bending, SEG Technical Program Expanded Abstracts, 453-457.
Waltham, D. [1988]. Two-point ray tracing using Fermat's principle. Geophysical Journal International, 93, no. 3, 575–582
Westwood, E., and Vidmar, P. [1987]. Eigenray funding and time series simulation in a layered-bottom ocean. The Journal of the Acoustical Society of America, 81, no. 4, 912-924.