AVO (amplitude variation with offset) modeling contributes significantly in seismic data acquisition design, and prestack seismic data processing and interpretation. It has become a common practice in prestack seismic analysis. It is also used for verifying and developing AVO theories. Conducting AVO modeling is, by nature, an exercise of multidiscipline integration. It thus enhances reservoir characterization and reduces the risk in hydrocarbon exploration.

This paper presents practical aspects of AVO modeling with the goal of better understanding and more effectively using AVO modeling in both prestack data processing and interpretation. Numerous examples demonstrate the applications of AVO modeling. We present approaches for generating, processing, and analyzing AVO synthetic data, and include the commonly used methodologies of single-interface modeling, single-gather modeling, 2D stratigraphic modeling, and 2D full-wave elastic-equation modeling. We illustrate the applications of these methods to P-wave data, converted-wave data, and elastic rock property inversion. In addition, we discuss some commonly encountered issues such as tuning and the effects of noise contamination in AVO processing and analysis. Finally, we demonstrate the role of AVO modeling in calibrating prestack seismic processing and in assisting data interpretation.

**Fundamentals in AVO modeling.**
Seismic rock properties are directly responsible for seismic wave propagation and seismic responses. They may be catalogued as basic rock seismic properties (P- and S-wave velocities and density, and \( V_p/V_S \) ratio and Poisson’s ratio), impedances, modulus rock properties (bulk modulus \( K \), shear modulus \( \mu \), Lamé’s constant \( \lambda \)), and anisotropic rock properties.

Often, rock properties and corresponding AVO responses can be discerned from well-log data. To demonstrate this, various rock properties were calculated using the log data from the Western Canadian Sedimentary Basin (WCSB) and are displayed in the plots in Figure 1, with the empirical relationships for shale (solid black line),

![Figure 1](image1.png) Seismic rock properties derived from a set of dipole sonic logs from the Western Canadian Sedimentary Basin. Data points of the gas sand, oil-saturated sand, and overlying and underlying shale are highlighted in red, green, black, and pink squares, respectively. Notice that rock properties in different domains have different sensitivity responding to fluid. The contrast between gas sand and overlying shale indicates a class 1 AVO response. These crossplots can be used as templates to interpret inverted elastic rock properties. The lines in black, blue, and red are the empirical relationships for shale, brine-saturated sand, and gas-charged clean sand.

![Figure 2](image2.png) Petrophysical analysis of a well from the Western Canadian Sedimentary Basin. Shale volume has direct effects on water saturation and porosity, which consequently results in changes in elastic seismic rock properties and seismic responses.
brine-saturated sand (solid blue line), and gas-charged clean sand (solid red line) overlain. Figure 1 demonstrates that the velocities and impedances do not provide sufficient discrimination as to whether the reservoir is gas-charged. However, the \( \frac{\text{VP}}{\text{VS}} \) ratio, Poisson’s ratio, \( \lambda \rho \), and \( \lambda \mu \) ratio do. Further, as expected, the data points of the oil-saturated sand (green squares) fall on the empirical line for the brine-saturated sand. In comparison with the gas sand, the lower impedance of the oil-saturated sand (red squares) indicates that it has higher porosity. The lithology contrast leads one to expect a class 1 AVO anomaly at the top of the gas sand because the impedance of the gas sand is significantly higher than that of the shale above it (black squares).

Petrophysical analysis is an important aspect in AVO modeling because the information can be used to assess reservoir conditions. The relationships between petrophysical and seismic rock properties can also be established and used in reservoir characterization. The petrophysical rock properties that directly relate to seismic rock properties are volume fractions of mineralogies, porosity, and water saturation (\( S_W \)). Fluid type, gas/oil ratio (GOR), oil and gas gravity (in API), and brine salinity (in ppm) form another set of important petrophysical properties. Fluid properties can be calculated based on derived or in situ measured pressure (\( P \)) and temperature (\( T \)). Predicting seismic rock properties, especially shear-wave velocity, is important in rock physics analysis. The steps to accomplish this usually include determining in situ conditions, calculating fluid properties, solid properties, and finally calculating seismic rock properties. The link between seismic rock properties (bulk modulus and shear modulus) and petrophysical rock properties (porosity, fluid type, water saturation, and mineral composition) can be seen in these Gassmann equations:

### Table 1. Simplified AVO equations.

<table>
<thead>
<tr>
<th>Shuey (1985)</th>
<th>( R(\theta) = R_p + G \sin^2 \theta + C (\tan^2 \theta - \sin^2 \theta) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solve for:</td>
<td>Assumptions and limitations: ( \text{VP} ) term is truncated for angles &lt; 25°. Change in gradient could indicate change in fluid content, but could also be caused by a change in lithology.</td>
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<tr>
<td>( R_p, G )</td>
<td>( \frac{\Delta \sigma}{(1-\sigma)^2} \sin^2 \theta )</td>
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<tr>
<th>Hilterman and Verm (1995)</th>
<th>( R(\theta) = R_p \cos^2 \theta + \frac{\Delta \sigma}{(1-\sigma)^2} \sin^2 \theta )</th>
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<tbody>
<tr>
<td>Solve for:</td>
<td>Assumptions and limitations: Based on Shuey’s equation. Assumes angles &gt; 30°. Assumes ( \frac{\text{VP}}{\text{VS}} = 2 ), and makes no density assumptions. Change in Poisson ‘reflectivity’ could indicate change in fluid content, but could also be caused by a change in lithology.</td>
</tr>
<tr>
<td>( R_p ), ( \frac{\Delta \sigma}{(1-\sigma)^2} ), or ( R_p )</td>
<td>( \frac{\Delta \nu}{\frac{\nu}{\nu'}} \left[ \frac{1}{2} \left( 1 + \tan^2 \theta \right) + g \left( 1 - 2 \left( \frac{\nu}{\nu'} \right)^2 \sin^2 \theta \right) \right] \left( \frac{\Delta \nu}{\frac{\nu}{\nu'}} \right) \left( \frac{\nu}{\nu'} \right)^2 \sin^2 \theta )</td>
</tr>
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</table>

### Smith and Gidlow (1987)

| \( \Delta \nu \frac{\nu}{\nu'} \), \( \Delta \nu' \frac{\nu}{\nu'} \) | \( \frac{\nu}{\nu'} \) assumes density follows Gardner’s equation. Fluid factor \( \Delta F \) based on velocity. Less than critical angle. Makes no assumption about \( \frac{\nu}{\nu'} \). |
| \( R(\theta) = \frac{1}{2} \left( \frac{\Delta \nu}{\frac{\nu}{\nu'}} \right) \left( 1 + \tan^2 \theta \right) - 4 \left( \frac{\nu}{\nu'} \right)^2 \left( \frac{\Delta \nu}{\frac{\nu}{\nu'}} \right) \left( \frac{\nu}{\nu'} \right)^2 \sin^2 \theta \) |

### Fatti et al. (1994)

| \( R_p, R_s \) | Assumptions and limitations: For angles < 50°. Requires no assumptions about \( \frac{\nu}{\nu'} \) and density. Fluid factor \( \Delta F \) based on impedance. |
| \( R_p, R_s \) | \( \frac{\Delta \lambda \lambda}{\lambda} \), \( \Delta \mu / \mu \) |

<table>
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<tr>
<th>Gray et al. (1999)</th>
<th>( \Delta \nu ), ( \Delta \nu' )</th>
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<tbody>
<tr>
<td>Solve for:</td>
<td>Assumptions and limitations: Makes no assumptions about ( \frac{\nu}{\nu'} ) and density.</td>
</tr>
<tr>
<td>( R(\theta) = \frac{1}{2} \left( \frac{\Delta \nu}{\frac{\nu}{\nu'}} \right) \left( 1 - \frac{\nu'}{\nu} \right) \sin^2 \theta \left( \frac{\Delta \nu}{\frac{\nu}{\nu'}} \right) \left( 1 + \frac{\nu'}{\nu} \right) \left( 1 - \frac{\nu'}{\nu} \right) \left( 1 - \frac{\nu}{\nu'} \right) )</td>
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where \( K_{\text{dry}} = \text{effective bulk modulus of the dry rock} \); \( K_{\text{sat}} = \text{effective bulk modulus of the rock with pore fluid} \); \( K_0 = \text{bulk modulus of mineral material making up the rock} \); \( K_{\phi} = \text{effective bulk modulus of the pore fluid} \); \( \phi = \text{porosity} \); \( \mu_{\text{dry}} = \text{effective shear modulus of the dry rock} \); and \( \mu_{\text{sat}} = \text{effective shear modulus of the rock with pore fluid} \). In Equation 1, \( K_0, \phi \), and \( K_0 \) are often calculated using petrophysical logs. Notice that Equation 2 indicates that fluid type and saturation do not affect shear modulus of rock.

Numerous empirical relationships for seismic rock properties, especially for shear sonic prediction, have been published. They are useful when either petrophysical properties are not available or rock physics theories do not address the complexity of rock properties. Commonly used empirical relationships are the P- and S-velocity linear relation for water-saturated clastics, also called the mudrock line (Castagna et al., 1985); the velocity-porosity-clay relation (Han, 1986); the critical porosity model for \( K_{\text{dry}} \) (Nur et al., 1995); the Greenberg-Castagna relation for mixed lithologies (Greenberg and Castagna, 1992); the velocity relation for clay-sand mixtures (Xu and White, 1996); and the empirical velocity relations for carbonates (e.g., Li and Downton, 2000). Often used petrophysical empirical relations are the density-velocity relation (Gardner et al., 1974); the sonic and porosity relation (Wyllie et al., 1956); and the resistivity-velocity relation (e.g., Faust, 1953). While the above
empirical relationships are useful, the empirical relationships derived locally are strongly recommended. Further, calibration using available well logs may improve rock property predictions.

Figure 2 shows a set of petrophysical log curves from a WCSB well. Shale volume ($V_{\text{sh}}$), effective porosity ($\phi_E$), and water saturation ($S_W$) were derived from resistivity, gamma ray (GR), density porosity, neutron porosity, and local petrophysical parameters. The observations from this petrophysical evaluation include:

1. Water saturation is proportional to $V_{\text{sh}}$.
2. Gas saturation increases with decreasing $V_{\text{sh}}$.
3. Porosity increases with decreasing $V_{\text{sh}}$.

Figure 3, based on this petrophysical analysis, shows how the volume of clay influences seismic rock properties. It is notable that clean sand is most sensitive to gas saturation since the distance between the clean sand data points and the empirical lines is greatest. The sensitivity or distance decreases with increasing shale volume. Further, similar to what is shown in Figure 1, the elastic rock properties of the oil-saturated sand again fall on the wet-sand empirical lines. This example shows that petrophysical analysis is useful for understanding the relationship between reservoir quality and seismic rock properties and, consequently, seismic responses.

AVO equations and attributes. To illustrate the relationship between rock physical properties and seismic reflections, we will use a two-layer interface with incident P-waves, reflected and transmitted P-waves, and converted S-waves as shown in Figure 4. The rock properties in the layers are P- and S-wave velocities, density, impedances, bulk modulus, shear modulus, Lamé's parameter, and the attenuation coefficient Q. The anisotropic properties are P- and S-wave velocity anisotropic parameters $\varepsilon$, $\lambda$, and $\delta$ (Thomsen, 1986). The rock properties listed above are of great interest in prestack seismic inversion. In AVO modeling, they are often obtained or derived from well-log data.

Two main steps of AVO modeling are to generate synthetic gathers, CMP gathers or common-image gatherers (CIG), by using the exact solution and then to extract AVO attributes by using approximate equations. AVO exact solutions are often generated by using the Zoeppritz equations with ray tracing and a plane-wave assumption, or by using the full wave elastic equation with a finite-difference method.

Aki and Richards (1980) simplified the Zoeppritz equations into a form that can be used to solve for meaningful reflectivities. The difference between the Zoeppritz exact

<table>
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<th>Table 3. AVO attributes and their relationships.</th>
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<tr>
<td>Rock interval property</td>
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<tr>
<td>------------------------</td>
</tr>
<tr>
<td>P-impedance</td>
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<tr>
<td>S-impedance</td>
</tr>
<tr>
<td>$\lambda$</td>
</tr>
<tr>
<td>$\mu$</td>
</tr>
<tr>
<td>Poisson's ratio ($\sigma$)</td>
</tr>
<tr>
<td>Gradient ($G$)</td>
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<tr>
<td>Fluid factor ($\Delta F$)</td>
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* $\gamma = V_p/V_s$; $b =$ slope of mudrock line
solution and the Aki-Richards’ approximation is small when zero-offset reflectivity << 1. Rearranged forms of the Aki-Richards’ equations are commonly used for describe and extracting AVO attributes. Shuey (1985) provides approximate P-wave AVO equations. Commonly used approximate P-wave AVO equations are Shuey (1985) for P-impedance reflectivity \( R_P \) and gradient \( G \); Smith and Gidlow (1987) for P- and S-velocity reflectivities \( R_{VP} \) and \( R_{VS} \); Fatti et al. (1994) for P- and S-impedance reflectivities \( R_P \) and \( R_S \); and Hilterman and Verm (1995) for P-impedance reflectivity \( R_P \) and Poisson’s reflectivity \( R_S \) (Table 1).

AVO equations for converted S-wave (e.g., Jin et al., 2000), a vertical transverse isotropic (VTI) medium (Thomsen, 1993), and a horizontal transverse anisotropic (HTI) medium (Ruger, 1996) are listed in Table 2. Using converted-wave data, density reflectivity and shear-wave velocity or impedance reflectivity can be calculated. An approach that solves P- and S-wave reflectivities and density reflectivity simultaneously is called simultaneous P-P and P-S inversion (e.g., Larson, 1999). Including both converted- and P-wave data in AVO inversion has the potential to obtain a more stable estimation of density reflectivity. Few efforts have been made to extract AVO attributes in VTI media because of the difficulties in simplifying anisotropic AVO equation from usually five to no more than three. In an HTI medium, Ruger’s two-term equation (for a single set of fractures) with an elliptical assumption of amplitude is often used and the attributes to be determined are fracture orientation and fracture density.

Commonly used AVO attributes and their relations are listed in Table 3. Some of these relations are exact and others approximate. Using Table 3, an AVO attribute can be derived from other AVO attributes by multiplying an operator to an attribute vector. For example, fluid factor can be derived from the ratio of dipole sonic logs. The dashed lines highlight the regional background trends, and the arrows show the direction in which the data points will shift when brine in rock is replaced by hydrocarbons. Figure 5a shows P-wave reflectivity \( R_P \) against S-wave reflectivity. Figure 5b, the crossplot of P-wave reflectivity \( R_P \) against gradient \( G \), is the one in which AVO types for a gas-charged clastic reservoir can be determined. In Figure 5d we can see that \( \Delta \lambda / \lambda \) and \( \Delta \mu / \mu \) have magnitudes approximately twice \( R_P \) or \( R_S \).

Figure 5c shows the plot of \( R_P \) against density reflectivity \( \Delta \rho / \rho \). This information is useful when density reflectivity is considered robust in discriminating lithologies or reservoirs. In general, the values of density reflectivities are much smaller in comparison to \( R_P \). To demonstrate this, we derived the equation \( \Delta R_P = f (\Delta V_P / V_P) - \Delta \rho / \rho \) (based on Gardner’s relation \( \rho = d V_P^2 \)). Since \( f \) has a value of about \( \lambda / 3 \), density reflectivity is about one quarter of P-wave velocity reflectivity.

Figure 6 shows a set of synthetic gathers corresponding to class 1–4 AVO responses from the top of a reservoir. The class 1 AVO response (higher-impedance gas sand) has a
Lithologies that do not follow the regional trend produce a peak corresponding to the top and base of the gas zone. P- and S-wave velocities are related. It shows that the data weighted difference between P- and S-wave velocities is called a fluid factor in P- and S-wave velocity crossplot space. The drop in P-wave velocity due to gas saturation produces a fluid-factor trough that corresponds to the top of the carbonate. Smaller changes in S-wave velocity produce a fluid-factor peak that decreases with offset. Fluid factor is a hydrocarbon interface indicator. It is the weighted difference between ∆V_p/V_p and ∆V_s/V_s or between R_p and R_s (Table 3). Figure 7 explains how fluid factor and P- and S-wave velocities are related. It shows that the data points from the gas sand, with lowered P-wave velocity due to gas saturation, deviate from the regional background trend. This results in a trough in fluid factor followed by a peak corresponding to the top and base of the gas zone.

Figure 6. Class 1–4 AVO responses to gas-charged clastic reservoirs.

Figure 7. Relationship between mudrock line, carbonate line, and fluid factor in P- and S-wave velocity crossplot space. The drop in P-wave velocity due to gas saturation produces a fluid-factor trough that corresponds to the top of the reservoir. In a clastic-carbonate environment, a fluid-factor peak corresponds to the top of the carbonate.

peak at zero-offset that decreases with offset and may change polarity at far offset. A class 2 AVO response (a near-zero impedance contrast) has a weak peak that decreases with offset with a polarity reversal at far offset, or a weak trough that increases in amplitude with offset. A class 3 AVO response (lower-impedance gas sands) has a trough at zero offset that brightens with offset. Finally, a class 4 AVO response (lower-impedance gas sands) has a trough that dims with offset.

Fluid factor is a hydrocarbon interface indicator. It is the weighted difference between ΔV_p/V_p and ΔV_s/V_s or between R_p and R_s (Table 3). Figure 7 explains how fluid factor and P- and S-wave velocities are related. It shows that the data points from the gas sand, with lowered P-wave velocity due to gas saturation, deviate from the regional background trend. This results in a trough in fluid factor followed by a peak corresponding to the top and base of the gas zone. Lithologies that do not follow the regional trend produce false fluid-factor anomalies. For example, reflection from a coal layer can produce a fluid-factor anomaly similar to that from a gas-sand reservoir. A carbonate layer encased by lower-impedance clastic rocks such as shale produces a peak at the top of the carbonate layer followed by a trough at the base (Figure 7). Ideally, a carbonate line should be used in calculating fluid factor for a porous and hydrocarbon-filled carbonate reservoir encased by tight carbonates. Further, when multilayer interbedded carbonate and shale are present, fluid-factor anomalies often become too complex to interpret. In this circumstance, an understanding of local geology becomes important to eliminate the false fluid-factor anomalies. In addition, a fluid-factor response may appear as a single peak or a single trough when seismic rock property contrast at the top or base of a reservoir is not strong enough to produce a fluid-factor anomaly.

In the following section we will demonstrate attribute inversion in AVO modeling and analysis by using a synthetic CMP gather. Figure 8 shows the example in which the low-impedance reservoir produces a class 3 AVO anomaly. We compared the AVO attributes extracted from the CMP gather to the attributes directly calculated from the well logs (and consequently called “reference” AVO attributes). The differences between the extracted attributes and the reference attributes are mainly due to offset-dependent tuning. NMO stretch, the angle range used in AVO extraction, and the approximations introduced in the simplified Zoeppritz equations.

Figure 8a shows angle-limited stacks. The stack with the angle range of 20–30° has a larger amplitude corresponding to the top and base of the reservoir. It also shows that the full-offset stack differs from the zero-offset stack (R_p). This explains why P-reflectivity extracted from CMP gathers rather than from a migrated structure (full-offset) stack should be used as input in impedance inversion. For this typical example, P-impedance inversion using the full-offset stack would underestimate the impedance and give an overly optimistic prediction of porosity. P- and S-reflectivities extracted using Fatti’s equation are shown in Figure 8b. The P-reflectivity has a trough corresponding to the top of the reservoir since the reservoir has low P-impedance. However, the S-reflectivity has a peak since the reservoir has high S-impedance. The fluid factor, as expected, exhibits a trough and a peak corresponding to the top and base of the reservoir.

The attributes extracted using Smith and Gidlow’s equation are shown in Figure 8c. ∆V_p/V_p and ∆V_s/V_s look similar to the R_p and R_s extracted by Fatti’s equation, telling us that the velocity-density relations may follow Gardner’s relation.

The gradient extracted using Shuey’s equation is shown in Figure 8d. The negative gradient corresponds to the reflection from the top of the reservoir, indicating an increase in amplitude with incident angle (trough brightening). The positive gradient corresponds to the base of the reservoir, also indicating an increase in amplitude with incident angle (peak brightening).

Figure 8e shows Poisson’s reflectivity extracted using Hilterman and Verm’s equation. At reservoir level, the Poisson’s reflectivity appears similar to the gradient and fluid factor; that is a trough followed by a peak at the top and base of the reservoir, respectively.

The last panel (Figure 8f) shows λ reflectivity, Δλ/λ, and μ reflectivity, Δμ/μ extracted using Gray’s equation. These reflectivities were scaled by 0.5 because their magnitudes are approximately twice P- or S-reflectivity. Note that Δλ/λ has a strong trough and peak corresponding to the top and
In practice, by examining the extracted AVO attributes as described above, one may determine which attribute(s) to use in defining a reservoir.

Tuning effects and NMO stretch. Tuning has a significant impact on seismic resolution and AVO responses. It may mask or induce an AVO anomaly. In AVO analysis, two types of tuning must be considered: tuning due to thickness of a reservoir and offset-dependent tuning due to amplitude variation with offset. Both affect the true AVO expression at an interface. In addition, offset-dependent tuning can be further affected by the differential traveltimes of adjacent reflections. To demonstrate offset-dependent tuning, we derived an equation to calculate the tuning effect on a primary reflection based on offset-dependent Ricker wavelets:

\[
R_p(\theta_i) = A_1 r_p(\theta_i) + A_2 r_i(\theta_i) \left[ 1 - 8\pi^2 \left( \frac{b}{L_0} \right)^2 \right] e^{-\pi^2 \left( \frac{b}{L_0} \right)^2}
\]

(3)

where \( A_1 \) and \( A_2 \) are the zero-offset reflectivities corresponding to two adjacent interfaces. Indices \( p \) and \( i \) represent primary reflection and the interfering reflection, respectively. \( \theta_i \) and \( \theta_p \) are the incident angles at the upper and lower interfaces; \( b \) is the thickness of the layer, and \( L_0 \) is the wavelength of dominant frequency. The tuning effects of a class 1 AVO and a class 3 AVO for a reservoir with \( V_p = 3674 \text{ m/s} \) and \( V_s = 2055 \text{ m/s} \) were calculated and shown in Figure 9. The lines in red and blue represent the tuning-free AVO responses from the top and base of the reservoir. The AVO responses with ratio \( b/L_0 \) equal to 1, 1/2, 1/4, and 1/8 are shown in Figure 9. A significant change in amplitude

Figure 8. Synthetic CMP gather and commonly used AVO attributes extracted from the gather: (a) angle stacks and full offset stack; (b) P- and S-impedance reflectivity (\( R_p \) and \( R_s \)); and fluid factor \( \Delta F \); (c) P- and S-velocity reflectivity (\( R_{vp} \) and \( R_{vs} \)); and fluid factor \( \Delta F \); (d) gradient \( G \); (e) Poisson’s reflectivity, and (f) \( \lambda \,- \) and \( \mu \,- \)refectivity (\( R_{\lambda} \) and \( R_{\mu} \)). The subscript \( r \) refers to the synthetic traces that were calculated directly from well logs.

Figure 9. Examples of offset (angle) dependent tuning calculated using Equation 3 and Shuey’s two-term AVO equation and Equation 3: (a) class 1 and (b) class 3 AVO responses. The lines in red and blue are AVO responses without tuning.
can be observed when $b/L_0$ equals $\frac{1}{2}$ and $\frac{3}{4}$. This illustrates that AVO analysis performed on reflectivities that suffer from tuning will be inaccurate.

To further demonstrate the tuning effect, two synthetic CMP gathers with a class 3 and class 1 AVO anomaly, respectively, were generated from well-log data. The class 3 AVO resulted from an unconsolidated sand reservoir that has an average P-wave velocity of 1900 m/s (Figure 10). The frequency bandwidth used was 8–14 Hz to 90–100 Hz. Due to the low impedance of the reservoir, the class 3 AVO anomaly can still be observed when the reservoir thickness is reduced to 5 m. The amplitudes extracted from the top of the reservoir show a significant tuning effect at a thickness of 1 m.

The class 1 AVO example is from a tight gas-sand reservoir that has an average velocity of about 4600 m/s (Figure 11). The bandwidth used was 10–15 Hz to 100–120 Hz. When the gas sand has a thickness of 18 m, the AVO response has a peak at near offset that dims with offset and reverses polarity at far offsets. The AVO response becomes a trough at zero offset and then dims with offset when the thickness was reduced to 10 m.

Notice that reducing frequency will result in a similar effect as tuning. Therefore, frequency content in a data set is essential in achieving an accurate AVO analysis.

NMO-corrected gathers may suffer significant differential moveout, which distorts the waveform and bandwidth especially at far offsets that correspond to large incident angles. In AVO processing, far-offset traces are often muted to limit the impact of NMO stretch. This imposes a constraint in the AVO attribute extraction that needs far-offset information, such as density reflectivity inversion using P-wave data. Researchers have been attempting to develop methods to combat NMO stretch. Swan (1997) developed a method by nulling NMO velocity error at a given two-way traveltime on a CMP gather. In addition, residual moveout due to anisotropy may also need to be corrected before AVO attribute extraction.

**Effects of noise contamination.** In AVO attribute extraction, CMP gathers or common image point gathers obtained through amplitude preserving processing are modeled by AVO equations to solve for AVO attributes. This is often conducted using angles of incidence and linear fitting of AVO amplitudes by L2 norm (the least squares) or L1 norm methods. The difference between L2 and L1 norms is that the former minimizes squared deviation while the latter minimizes the absolute deviation of the data. L1 norm is robust in eliminating coherence noise such as multiples. Ferré et al. (1999) proposed an approach to optimize AVO attribute extraction by using the least median of squares method (Walden, 1991). This method is similar to L1 norm method but with a progressive converging process to achieve an optimized solution.

Signal-to-noise ratio, fold of a CMP gather, and offset range all affect the results of an AVO analysis. Offset-dependent attributes such as S-reflectivity and gradient are most influenced by these factors. In investigating the effects of noise, random and/or coherent noise can be added to synthetic gathers. A CMP or CIP gather with random and coherent noise can also be generated using elastic wave-equation modeling. Figure 12 shows an example in which random noise was added in a CMP gather with a signal-to-noise ratio of 1:0.2. AVO extraction used Fatti’s equation. As expected, noise has little effect on the estimation of P-reflectivity. The estimated S-reflectivity at reservoir level, however, experiences significant distortion.

**AVO modeling methodologies.** The Zoeppritz equations or the Aki-Richards equation with ray tracing, and full elastic wave equation with the finite-difference method (FDM) are commonly used to generate synthetic CMP or CIP gathers. The former has the advantage of being fast and easy for analyzing primary reflections. Elastic wave-equation modeling calculates particle displacements in the subsurface and accounts for direct waves, primary and multiple reflection waves, converted waves, head waves, and diffractions. It overcomes the shortcomings of ray tracing which can break down in many cases, such as at edges where the calculated amplitude is infinite or in shadow zones.

Different AVO modeling methodologies, based on the Zoeppritz equations with ray tracing and elastic wave equations, have been developed to take into account issues associated with data acquisition, processing, and interpretation. The most commonly used methods are single-interface modeling, CMP gather modeling, 2D stratigraphic modeling,
and 2D full wave elastic modeling. 3D elastic wave-equation modeling needs significant computer power and currently can only be performed by some major oil companies and a few research institutions.

**Single interface modeling.** Single-interface modeling is free from tuning. It is often used to show theoretical AVO responses. To demonstrate this method, two examples were generated. Figure 13a shows the class 3 AVO responses from an interface of shale overlying porous gas sand, where two- and three-term Shuey’s equations are compared to evaluate the error introduced by truncation of the third term. Also, the maximum and minimum amplitude locations and inflection point location of the three-term equation were calculated using the first and second derivatives. It can be seen that the error due to truncation of the third term increases at large incident angles. A second example shows the amplitude variation with both offset and azimuth for an HTI medium or for the rock with vertical fractures (Figure 13b). In the fracture plane, the rock is considered isotropic and the AVO response is the same as that from isotropic medium. In the plane perpendicular to the fracture plane, the AVO has a different response. By fitting an amplitude surface to the data using Ruger’s equation (1996), the orientation of the fractures can be solved, and “fracture density” can be calculated by using the gradients parallel and perpendicular to the fractures.

**Single-gather modeling.** For single CMP gather modeling, the Zoeppritz equations with ray tracing and/or the full wave elastic equation with finite-difference method are often used. While Zoeppritz equation modeling calculates primary reflections, elastic modeling accounts for transmission losses, multiples, and converted waves. Therefore, the Zoeppritz equation with ray tracing and elastic wave equation modeling are often conducted at the same time. Since the modes of an elastic wave cannot be isolated from each other in elastic wave-equation modeling, one may use Kennett’s method (1979) to calculate a single mode of elastic waves such as multiples or converted waves. Other approaches to conduct wave-equation modeling can be found in Carcione et al. (2002).

Figure 14 shows the CMP gathers generated by the Zoeppritz equation with ray tracing and elastic wave-equation modeling with finite difference method. It can be seen that the Zoeppritz modeling produces clean primary reflections (Figure 14a), and the elastic wave-equation modeling generates more waveforms including interbed multiples and converted-wave reflections (Figure 14b). The elastic
modeling may be used to test strategies in noise attenuation. In practice, acoustic wave-equation modeling may be incorporated with elastic wave-equation modeling to identify converted-wave energy.

Finally, we present an example of converted-wave AVO modeling (Figure 15) which is useful in analyzing multi-component data. In practice, three-component data are separated to P-wave (P-P) data and converted-wave (P-S) data for AVO processing. Often, converted-wave data have lower frequency bandwidth. The P-P data can be incorporated with P-S data in AVO attribute inversion. The CMP gathers in Figure 15 were generated using P-P and P-S Zoeppritz equations with ray tracing, where the traveltme of the converted-wave gather was corrected to P-wave time. The P- and S-reflectivities were inverted from the P-P gather, and inverted S-reflectivity and density reflectivity were inverted from the P-S gather.

Figure 15. P-wave and converted-wave AVO modeling using Zoeppritz equations with ray tracing; the two-way traveltme of the converted-wave gather was corrected to P-wave time. The attributes shown are the full-offset stacks of the P-P and P-S gathers, inverted P- and S-reflectivities (RPpp and RPps) from the P-P gather, and inverted S-reflectivity and density reflectivity (RPss and RPps) from the P-S gather.

the frequency bandwidths of 10–14 to 110–120 Hz and 10–14 to 55–60 Hz were used in the P-P and P-S AVO modeling, respectively.

Two-dimensional stratigraphic modeling. 2D stratigraphic modeling takes into account lateral variations which can be due to structure, reservoir thickness, porosity, lithology, fluid type, and fluid saturation. The velocity and density models may come from seismic interpretation or constructed using well logs as control points. Using the velocity and density models as input, CMP gathers of a 2D line can be generated and then processed.

Figure 16 shows 2D stratigraphic modeling based on the wells from a play in the Gulf of Mexico, where the gas zones have low P- and S-impedances. It can be seen that P-reflectivity brightens at the gas well location but does not provide definitive information for defining the reservoirs; and (b) fluid-factor stack shows anomalies with a trough and a peak corresponding to the top and base of each reservoir. The overlying logs P-impedance on P-reflectivity, and Vp/Vs ratio on fluid-factor stack.

Figure 16. 2D stratigraphic AVO modeling for a play with several low-impedance gas zones in the Gulf of Mexico: (a) P-reflectivity brightens at the gas well location but does not provide definite information for defining the reservoirs; and (b) fluid-factor stack shows anomalies with a trough and a peak corresponding to the top and base of each reservoir. The overlying logs P-impedance on P-reflectivity, and Vp/Vs ratio on fluid-factor stack.

In this example, we inverted P- and S-reflectivities from the synthetic CMP gathers, and then performed P- and S-impedance inversion and calculated λρ and μρ. Different sensitivity of the inverted elastic rock properties in response to the gas

Figure 17. Inverted elastic rock property sections from a 2D stratigraphic AVO modeling for a gas-charged reservoir: (a) P-impedance, (b) S-impedance, (c) λρ, and (d) λρ−μρ. P- and S-impedances do not provide defining information on the reservoir. λρ and μρ, however, show a clear anomaly.

A second example is from a WCSB gas play with well control at four locations (Figure 17). The target zone varies from silt sand to gas sand, and then to brine-saturated sand. In this study, we inverted P- and S-reflectivities from the synthetic CMP gathers, and then performed P- and S-impedance inversion and calculated λρ and μρ. Different sensitivity of the inverted elastic rock properties in response to the gas

Figure 17. Inverted elastic rock property sections from a 2D stratigraphic AVO modeling for a gas-charged reservoir: (a) P-impedance, (b) S-impedance, (c) λρ, and (d) λρ−μρ. P- and S-impedances do not provide defining information on the reservoir. λρ and μρ, however, show a clear anomaly.

Figure 14. Single CMP gather AVO modeling for a well in the Western Canadian Sedimentary Basin: (a) Zoeppritz equation with ray tracing, and (b) elastic wave-equation modeling with finite-difference method. The former generates primary-only reflections, and the latter generates primary reflections, multiples, and other modes of reflections such as converted waves. The elastic modeling shows the distortions in some of the primary reflections. The interbedded multiples may manifest as real reflections.

Figure 14. Single CMP gather AVO modeling for a well in the Western Canadian Sedimentary Basin: (a) Zoeppritz equation with ray tracing, and (b) elastic wave-equation modeling with finite-difference method. The former generates primary-only reflections, and the latter generates primary reflections, multiples, and other modes of reflections such as converted waves. The elastic modeling shows the distortions in some of the primary reflections. The interbedded multiples may manifest as real reflections.
Figure 18. Elastic wave-equation modeling with finite-difference method for a structurally complex play in Mackenzie Delta, Canada: (a) input P-wave velocity model, (b) a shot gather, (c) structure stack, and (d) prestack depth-migrated section. The location of the shot gather is indicated in (c) and (d).

reservoir is evident: The P- and S-impedances are unable to delineate the reservoir, but the $\lambda \rho$ and $\lambda \rho - \mu \rho$ attributes show a clear gas anomaly.

Two-dimensional elastic wave-equation modeling. In structurally complex areas, seismic imaging alone can not completely describe a reservoir. Therefore, there is great interest in using 2D or 3D elastic wave-equation modeling to generate synthetic data for AVO analysis. In comparison to ray-tracing methods, elastic wave-equation modeling generates more realistic synthetic data. Recent advances in computing power make single-shot gather modeling trivial and 2D modeling practical.

Figure 18 shows the application of 2D full wave-elastic modeling with finite-difference method to a structurally complex area in the Mackenzie Delta, Canada. The purpose was to investigate whether the reservoir can be characterized by AVO attributes extracted from prestack depth migration. In this study, the velocity models and density model were constructed from well logs. Figure 18a shows that an unconformity is between 1750 m and 2700 m, and the gas reservoir is under the unconformity and sealed by a fault. Figures 18b–d display a shot gather, the structure stack, and the prestack depth-migrated section. The results from the AVO analysis are shown in Figure 19. Figures 19a and 19b illustrate the conversion of a common image gather from the P- and S-reflectivities extracted from the common image gathers.
clearly defined. This example demonstrates that, for a structure play, an a-priori study using elastic wave-equation modeling and prestack depth migration helps determine whether AVO analysis is feasible.

AVO modeling applications. AVO modeling contributes significantly to prestack data processing and interpretation. Inappropriate processing may be detected by examining AVO responses in CMP gathers or by analyzing inverted AVO attributes. An understanding of local petrophysics, rock physics, and geology provides constraints for QC in AVO processing and interpretation. Ideally, AVO processing and interpretation should be conducted in parallel with AVO modeling. As a result, uncertainty and risk can be reduced.

Data processing. Calibration on CMP gathers and AVO
attributes by AVO modeling is useful in optimizing AVO processing. It is often implemented by using well logs, synthetic gathers, walkway VSP data, and known relationships between AVO attributes or between seismic rock properties. Calibration can be performed at a CMP location, or globally, on a data set to answer such questions as these:

- Have the CMP gathers been properly processed with an amplitude-preserving processing workflow?
- Are phase, tuning, signal-to-noise ratio, and frequency bandwidth influencing the AVO solutions?

Tying synthetic CMP gather(s) to recorded seismic data often gives a quick insight into data quality and to type of AVO response. One may perturb the well logs to represent possible reservoir conditions to examine the variation of seismic responses. For example, gas substitution may be performed on a wet well to examine the gas effects. Other parameters often perturbed in AVO modeling are porosity, reservoir thickness, lithology, and frequency and angle to be used for AVO attribute extraction.

Figure 20a shows an example in which a synthetic CMP gather is tied to a recorded CMP gather from a 2D survey. By comparing the AVO responses from the top of the gas reservoir, a class 1 AVO anomaly with polarity reversal at the far offsets is confirmed. This is further validated quantitatively by the amplitudes extracted from both the seismic and the synthetic gathers (Figure 20b). In this study, 3D data were initially considered for AVO processing. Further study, however, revealed that the 3D data have a frequency content below the resolution for delineating the reservoir. Figure 21a shows that a synthetic CMP gather generated using the bandwidth from the 2D data ties to the 3D data. It is evident that the frequency content in the 3D data is significantly lower than that in the 2D data. In Figure 21b, a good well-tie was reached when the frequency bandwidth of the 3D data was used in the synthetic CMP gather.

Calibration can be applied to other AVO attributes such as S-reflectivity, gradient, and fluid factor. It can also be applied by plotting P-reflectivity against S-reflectivity, and P-impedance against S-impedance. Figure 22 shows an example using well logs to calibrate inverted elastic rock properties in a gas-charged dolomite play. In Figures 22a–b, the inverted elastic rock properties from the zone of interest are highlighted in black squares. The data points shift toward low \( k \rho \) values and low \( k/\mu \) ratios, indicating effects of gas. To calibrate this, dipole sonic logs from the same area were plotted (Figures 22c–d). In this well, the data from the gas-charged dolomite (red squares) have lower porosity and the data from a brine-saturated porous dolomite (green squares) have higher porosity. The comparison between seismic data and well-log data confirms that the reservoir is gas-charged because it has low values of \( k \rho \) and \( k/\mu \). Also, it indicates that the reservoir has porosity similar to the data points highlighted by the green squares in Figures 22c–d.

Global calibration using AVO modeling is an approach to conduct QC for a prestack seismic data set. For example, AVO within a time window or from a specific reflection can be compared and calibrated with the results from AVO modeling. This type of exercise can tell whether the data were processed by an amplitude-preserving workflow.

Phase analysis is an important aspect in AVO processing QC. Non-zero-phase data result in inaccurate AVO attributes and inverted elastic rock properties. Phase analysis is usually accomplished by crosscorrelation using zero-offset synthetics and migrated stack. It produces wavelets accompanied by phase information. Well ties using CMP gathers may remove some ambiguity because of additional information from AVO responses. For example, a typical AVO response or an AVO anomaly can be criteria to tie well to seismic. Also, improved wavelet(s) may be obtained by using extracted P-wave reflectivity \( R_P \) instead of migrated stack because the former are considered zero-offset data. Furthermore, AVO attributes such as fluid factor may be useful in phase analysis. For example,
in a clastic environment, a peak followed by a trough in a fluid-factor section may indicate that the seismic data have reversed polarity.

**Data interpretation.** AVO modeling has an important role in assisting interpretation on CMP gathers, AVO attributes, and inverted elastic rock properties. It helps validate AVO responses and links seismic expression to known reservoir conditions. It is able to integrate petrophysics, rock physics, and geology with seismic data. As a result, confidence in interpretation is increased, and the risk in drilling is reduced.

Tuning effects may invoke or mask an AVO anomaly as described earlier in this paper. AVO modeling using varied frequency bandwidth or reservoir thickness may yield the answers. Complex lithologies may produce AVO anomalies that can be investigated by AVO modeling as well. For example, AVO responses from tight sand may manifest as class 1 AVO anomalies and show brightening of amplitude in a migrated stack. This type of AVO anomaly may be excluded by high P- and/or S- impedances. Coal, carbonate, and the lithologies that do not follow the mudrock trend may produce fluid-factor anomalies. It is possible to exclude these anomalies through integrating geologic information. Further, high clay content in rock results in low gas saturation, and this type of partial gas saturation may be distinguishable because the increase in clay content results in an increased $V_p/V_S$ ratio, and consequently changes the AVO responses.

In data interpretation, synthetic CMP or CIP gathers should be processed through the same data processing sequence used for field data. The synthetic gathers, AVO attributes and inverted elastic rock properties can thus be directly compared to their counterparts from seismic data.

As a special case study, Figure 23 shows an example that uses elastic wave-equation modeling to investigate the interference between primary reflections, multiples, and converted energy at the Wabamun dolomite porosity in the WCSB. This is a classic case in which multiples were often misinterpreted as porosity on migrated stack. We were interested in knowing how multiples and converted energy interfere with the primary reflections at the reservoir level. AVO modeling was conducted with and without porosity. Figure 23 shows that there is a significant difference in seismic responses between these two cases. Further, the information from this study helped optimize noise attenuation and amplitude recovery.

The final example is from a study on a carbonate reservoir in the WCSB (Figure 24). AVO modeling shows that the gas-charged dolomite reservoir that has an average porosity of about 14% produces a class 3 AVO anomaly. This agrees with the AVO response observed in the CMP gather at the producing well. In contrast, a completely different seismic response was observed at the tight well locations. In this case, the information obtained from AVO modeling validated the seismic responses.

**Conclusions.** We have demonstrated the importance and practical aspects of AVO modeling in prestack seismic processing and interpretation. We have also presented and dis-
cussed the AVO modeling methodologies—single-interface modeling, single-gather modeling, 2D stratigraphic modeling, and 2D full wave elastic modeling. Except for the AVO modeling using Zoeppritz equation with ray tracing, full wave elastic-wave equation modeling helps AVO analysis in structurally complex areas, and converted-wave AVO modeling provides useful information on analyzing converted-wave data. We have demonstrated that it is essential to examine seismic rock properties and their contrasts in order to understand the sensitivity of seismic rock properties responding to fluid. The information from analyzing petrophysical and seismic rock properties is useful in predicting AVO responses. In AVO modeling, effects of reservoir thickness, sensitivity of rock properties responding to fluid and lithology are important and need to be investigated. Special attention needs to be paid to seismic resolution as it alters AVO responses. Offset-dependent tuning, noise contamination, NMO stretch, limitations in seismic data acquisition, and structural effects need to be studied. Furthermore, AVO modeling is an exercise in multidisciplinary integration of petrophysics, rock physics, seismic, geology, and petroleum engineering. It provides important information on reservoir characterization and risk reduction in hydrocarbon exploration. Finally, AVO modeling of an anisotropic medium, especially an HTI medium, faces challenges due to the difficulties in obtaining accurate anisotropic input.


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