A way from the wells, interpolation methods are used to populate reservoir models with petrophysical properties such as porosity and permeability. These methods proceed by estimating the property values at a given location in the reservoir. That estimation depends on how strongly the rock material at the location of interest is correlated to surrounding data. Interpolation techniques use measures of spatial correlation, inferred from observations that are functions of the ‘deposition’ distance between two (or more) locations. However, today, distances between sediment particles are very different from what they were at the time these particles were deposited. In order to approximate original distances, algorithms rely on transformations implied by the geometry of ‘stratigraphic’ grids used to represent 3D subsurface reservoir models. These transformations map the grids into regular Cartesian grids of equal cell dimensions assumed to be the space of deposition or ‘parametric’ space.

**Stratigraphic distances and the parametric space**

The location of any particle of sediment ‘s’ observed today in a reservoir can be characterized in two ways:
- Firstly, by the Euclidean coordinates (x, y, z) of its location today, where (z) is the elevation of ‘s’ and (x, y) are its geographic coordinates in the 3D geological space;
- Next, by its ‘paleo-coordinates’ (u, v, t), where (t) was the deposition time of ‘s’ and (u, v) were its paleo-geographic coordinates at depositional time (t).

To accurately populate reservoir models with petrophysical properties, interpolation methods (geostatistics, inverse distance) require computation of ‘stratigraphic distances’ d(a, b) between any pair of particles of sediment (a, b) located today at positions r(a) and r(b) in the reservoir. Theoretically, d(a, b) should be computed as follows:

\[
d(a, b) = \sqrt{(u(b) - u(a))^2 + (v(b) - v(a))^2 + \lambda \times (t(b) - t(a))^2}
\]

Where \([u(a), v(a), t(a)]\) and \([u(b), v(b), t(b)]\) are the paleo-coordinates of (a) and (b), respectively, and \(\lambda\) is a scaling factor which accounts for how correlations range vary throughout geological time.

Generally, in commercial reservoir modelling applications, the (i, j, k) indexes of the hexahedral (cubic) cells of the structured stratigraphic grid that represents the reservoir, are used in place of the (u, v, t) paleo-coordinates, to approximate the stratigraphic distance \(d(a, b)\) by \(D(a, b)\), as follows:

\[
D(a, b) = \sqrt{\alpha \times (i(b) - i(a))^2 + \beta \times (j(b) - j(a))^2 + \gamma \times (k(b) - k(a))^2}
\]

Where \(\{i(a), j(a), k(a)\}\) and \(\{i(b), j(b), k(b)\}\) are the indexes of the two cells containing the particles of sediment (a) and (b), respectively, and \((\alpha, \beta, \gamma)\) are the sizes of the cells in the (i), (j) and (k) directions, respectively.

For \(D(a, b)\) to equal \(d(a, b)\), two constraints must be honoured:
- **Constant cell sizes**: the size of cells of the stratigraphic grid must be constant
- **Translation invariance**: the variations \(\Delta i, \Delta j, \Delta k\) of the indexes between two cells must correspond to the same variations \(\Delta u, \Delta v, \Delta t\) of the paleo-coordinates whatever the position of these cells in the reservoir. In other words:

\[
\Delta u = \alpha \times \Delta i \quad ; \quad \Delta v = \beta \times \Delta j \quad ; \quad \Delta t = \gamma \times \lambda \times \Delta k
\]

If these constraints are satisfied, the \((u, v, t)\)-paleo-coordinates and the \((i, j, k)\) indexes can then be used equivalently to compute the distances required by interpolation algorithms. It can be observed that:
- \((u, v, t)\) or \((i, j, k)\) define a *parameterization* of the subsurface which associates each \((x, y, z)\) location in the

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In a first type represented in Figure (2)-A, the cells are still constrained to have hexahedral shapes. To preserve a cubic shape, the faults surfaces are then approximated by ‘stair-steps’.

In a second type represented in Figure (2)-B, the cells are no longer constrained to have hexahedral shapes. These grids are generated using a ‘cookie-cutter’ algorithm consisting of moving vertically a ‘cutting’ horizontal 2D grid whose intersections with the horizons and faults generate polyhedral cells. We call these grids XYT grids.

These new grids were primarily designed for reservoir flow simulation purposes. To that end, and as required by most commercial flow simulators, the computation of the transmissibility assigned to the ‘pipe’ connecting the centres of two adjacent cells must be, as much as possible, orthogonal to the common face shared by these cells:

- In a first type represented in Figure (2)-A, the cells are still constrained to have hexahedral shapes. To preserve a cubic shape, the faults surfaces are then approximated by ‘stair-steps’.
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Dangerous side effect of pillar-based grids

Modelling practitioners and reservoir engineers commonly build pillar based (i, j, k) stratigraphic grids, whose columns of cells in the (k) direction never cross the faults. In the neighbourhood of non-vertical faults, this leads to the generation of hexahedral cells of non-constant size and non-cubic shape. Figure (1) illustrates a vertical cross section through a reservoir modelled with a pillar grid. Because of the distortions induced by variations in cells-size, the sand bodies although of equal size in depositional space do not have the same dimensions in today’s (x, y, z) geological space. In comparison a grid generated using the UVT Transform (or UVT grid) shows no distortion of volumes.

In such a case, the actual volume of the reservoir rocks will be either under-estimated or over-estimated depending on their locations relative to the non-vertical faults. Such a bias can have a dramatic impact on the estimation of both in-place volumes and associated reserves, as variations of the lateral and vertical extend of the sand bodies will affect the connectivity of the reservoir, thereby impacting the flow of fluids through the reservoir.

New generations of grids

To avoid one of the drawbacks of a pillar approach described above, several new types of grids have been introduced in which the columns of cells in the (k) direction are allowed to cross faults. For example:

Figure 1 (A) vertical cross section in pillar based grid and in a UVT Grid (B). The dimensions of the sand bodies are distorted in the pillar based grid.
In the case of stair-step grids, the orthogonality pipe/face is explicitly ensured everywhere, even in the neighbourhood of faults: as a consequence, transmissibility can be computed properly.

In the case of ‘XYT’ grids, the orthogonality pipe/face is explicitly ensured everywhere, but in the neighbourhood of faults. If a pipe is cut by faults, to compute transmissibility, the common face (fault) can be replaced by its projection onto a virtual plane, orthogonal to the pipe: note that using such an implementation trick is equivalent to building a ‘virtual’ stair-step grid.

It should be noted that, contrary to stair-step grids which have quasi-cubic cells everywhere, in the neighbourhood of faults cookie-cutter grids generate complex polyhedral cells which may induce numerical problems in flow simulators.

Other types of grids with analysis of their pros and cons are presented in more detail in Mallet (2008).

Dangerous side effect of non-active and dead-cells
In the presence of non-vertical faults, any (i, j, k) indexing of cells can lead to severe distortions in the calculation of stratigraphic distances. This limitation, observable with all types of grids, often results in modelling errors which can have drastic consequences in the evaluation of recoverable volumes.

Index coherency test (ICT)
When using a grid with an (i, j, k)-indexing of cells, one can easily test if this indexing can be used for geostatistical applications. As illustrated in Figures 3 and 4, one can proceed as follows:

1. Build a grid with horizontal horizons and a non-vertical fault cutting the entire domain with a dip of 45 degrees and with a throw of about 1/3 of the vertical extent of the grid;
2. Choose a cell b, neighbouring the non-vertical fault (in the depositional space);
3. Choose a value ‘n’ such that the cell with indexes {i(b) ± n}, {j(b) ± n} and {k(b) ± n} still belongs to the grid;
4. In each cell a of the grid, store the value D(a, b) computed using Equation (2) with parameters ‘α’, ‘β’, ‘γ’, and ‘λ’ set equal to 1; and then,
5. Display a region of the volume for which D(a,b) is lower or equal to ‘n’.

For ‘n’ large enough, the value V(n) so obtained should be close to the volume of a sphere with radius ‘n’. If this is not the case, then this grid should not be used for geostatistical property modelling purposes. Figure 3 and 4 shows the ICT Test done on both XYT grids and UVT Grids.

Introducing the UVT-transform
As demonstrated above, in presence of faults not-orthogonal to layers, there is no possible (i, j, k) indexing allowing stratigraphic distances to be computed with Equation (2). Therefore, the only solution to overcome this problem is to use Equation (1) and compute the distance d(a, b) as a function of the paleo-coordinates of the centres (a) and (b) of any pair of cells. For that purpose, it is necessary to determine the functions u(x, y, z), v(x, y, z) and t(x, y, z) transforming any point (x, y, z) of the geological space into a point (u, v, t) in the parametric (depositional) space.

\[
(x, y, z) \rightarrow (u, v, t)
\] (4)

Such a transformation is called a UVT-transform. Assuming a minimal deformation structural tectonic style, it can be shown that the functions u(x, y, z), v(x, y, z), and t(x, y, z)
must honour, as much as possible, the following system of coupled non-linear differential equations where \((\text{grad } f)\) stands for the gradient of \(f(x, y, z)\) and the dot symbol ‘.’ stands for the dot product of vectors:

\[
\begin{align*}
1) \quad \text{grad } u \cdot \text{grad } v &= 0 \\
2) \quad \text{grad } t \cdot \text{grad } u &= 0 \\
3) \quad \text{grad } t \cdot \text{grad } v &= 0 \\
4) \quad ||\text{grad } u|| &= ||\text{grad } v|| = 1
\end{align*}
\]

To ensure the geological consistency of the models, optional constraints can be added, for example to take into account the tectonic style of the faults (e.g., normal or reverse). For an in-depth presentation of the underlying concepts of the UVT-transform, the reader is referred to (Mallet, 2004) and (Mallet, 2008).

- There is an infinity of possible functions \(u(x,y,z), v(x, y, z),\) and \(t(x, y, z)\) honouring the constraints defined by Equation (5) and the following data must be specified to remove any ambiguity:
  - Sets of points \([S(t_1), S(t_2), \ldots]\) located on horizons \([H(t_1), H(t_2), \ldots]\) must be provided with their associated geological times \([t_1, t_2, \ldots]\) sorted in such a way that \(H(t_i)\) was deposited at geological time \(t_i;\)
  - A reference point \((x_0, y_0, z_0)\) must be chosen as the origin of the \((u, v)\) paleo-coordinates:
    \[
u(x_0, y_0, z_0) = v(x_0, y_0, z_0) = 0\]
  - The direction of \((\text{grad } u)\) at the reference point \((x_0, y_0, z_0)\) must be specified.

Figure 5 shows an example of UVT-transform in the presence of a complex geological structure with so called ‘X’, ‘Y’, and ‘λ’ faults:

- In the parametric (depositional) space, the images of the horizons are perfectly horizontal planes and the faults have disappeared.
- The reverse UVT-transform maps any sugar-box of the depositional space into a set of hexahedral cells in the geological space: note that these cells may be split by the faults.
- In the geological space, the \(u(x, y, z), v(x, y, z),\) and \(t(x, y, z)\) functions are discontinuous across the faults, and, for any geological time \(t_i\), the iso-value surfaces \(t(x,y,z)=t_i\) represents the horizon \(H(t_i)\). Consequently, \(t(x, y, z)\) virtually models an infinity of horizons which can be extracted for any geological time.
- As a result of the Equations (5-1), (5-2), and (5-3) in the geological space and for any paleo-geographic coordinates \((u_i, v_j)\), the iso-value surfaces \(u(x, y, z) = u_i\) and \(v(x, y, z) = v_j\) are mutually orthogonal and are also orthogonal to the horizons.
- As a result of Equation (5-4), the structural distances along the horizons are not distorted.

The last two point above are the most important because they justify the use of the \((u, v, t)\) parameters as defined, to compute correct stratigraphic distances without using any grid, and therefore, without requiring the artificial introduction of non-active cells.

**Using the UVT-transform**

The UVT-transform can be used in a multitude of applications. For example:
In place of the classical (i, j, k) indexes, each cell of a grid covering the geological domain is assigned the (u, v, t) paleo-coordinates of its centre. This allows geostatistical methods to be applied on any type of grid, even with highly unstructured polyhedral grids.

As Figure 5 suggests, the depositional space can be covered with a regular rectilinear grid G* parallel to the (u), (v), and (t) axes and proceed as follows to model the static petrophysical properties of the reservoir:

a. Use the direct UVT-transform to ‘transport’ the data observed on well logs from the geological space to the depositional space;

b. Perform realizations of the geostatistical model on the grid G* covering the depositional space;

c. Use the reverse UVT-transform to ‘paint’ directly the geological space, or any cell of a grid G in the geological space, with the properties generated on the grid G* (e.g., see Figure 6).

**UVT-transform versus 3D balanced unfolding**

The image of all the horizons in the (u, v, t) parametric space being horizontal planes, considering the image of the reservoir in the parametric domain as a 3D balanced unfolding is tempting. There are two major differences between UVT-transform and 3D balanced unfolding:

- First, in the (u, v, t) parametric domain, contrary to a restored geological model, the vertical dimension (t) is not a length but a geological time: as a consequence, all the horizons in the parametric domain are parallel horizontal planes (see Figure 5).

- Second, in the parametric domain, all the horizons are flattened in one go: this is different from restoration techniques where horizons are flattened one at a time.

**Practical aspects of the UVT-transform**

From a theoretical point of view, the UVT-transform provides the most rigorous support on which to perform geostatistical reservoir property modelling with currently commercially available algorithms. Implemented as the underlying foundation in the Paradigm SKUA Suite, the UVT-transform has removed traditional reservoir modelling barriers by enabling a true 3D solution and therefore guaranteeing a coherent integrated representation of the earth from seismic and geological interpretation to flow simulation.

Other notable benefits of using the UVT-transform include:

- Facilitating the construction of complex reservoir structures including any faulting configuration, stratigraphic style, unconformities, and specifically extremely thin reservoir intervals.

- The use of all types of interpretation (seismic and geological) data. UVT-transform based models do not require any
simplification or removal of the data, which can represent an important part of any reservoir study.

- The ability to construct models with very limited user interaction ('One click modelling') yet enabling for constraints rarely available or reliable such as vertical fault throw and strike-slip displacement information.
- Enabling the accurate construction of flow simulation grids with stair-step representation of the fault and the exact mapping of reservoir properties during the upscaling which is required to go from a geostatistical support to a flow simulation support. These types of grids allow keeping the integrity of the reservoir structure yet are optimal for numerical modelling of fluid flow (Gringarten et al., 2008).

Case study examples

1 - The complex area of Figure 7 is located in a regional basin that underwent trans-tensional, sinistral shear during the rift phase that reactivated later as a transpressional, dextral shear.

Constructing a full reservoir model for this particular field is impossible using traditional pillar technology. Even though the model only contains a limited number of seismi-

Figure 6 Application of UVT-transform to (A) the modelling of stratified reservoirs (UVT grids) and (B) permeability. Black lines represent the intersection of (u) and (t) with the vertical cross-section: note the regularity of the (u, v, t) parameterization which guarantees a correct computation of stratigraphic distances. (Courtesy of Paradigm SKUA software).

Figure 7 (a) Complex fault rift system, (b) associated geological model containing the complete overburden and under burden.
To enable robust geostatistical modelling of reservoir properties, a new technology, called the UVT-transform, is introduced. It allows the construction of geological structures, whatever their complexity, while honouring all available data, and ensures a coherent mapping between the location of sediment particles today and their location in the original space of deposition, the parametric space. This methodology provides a true 3D approach and opens new avenues for multiples aspects of subsurface reservoir modelling.

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