Tu N102 14

Fourth-order NMO Velocity for Compressional Waves in Layered Orthorhombic Media

Z. Koren* (Paradigm) & I. Ravve (Paradigm)

SUMMARY

We derive azimuthally dependent fourth-order effective velocity and moveout parameters for compressional waves propagating in a layered model that consists of orthorhombic layers. The subsurface layered medium is considered as a locally 1D model, where each layer can be characterized by orthorhombic parameters. The orthorhombic layers have a common vertical axis but different azimuthal orientations of horizontal axes. For a 1D vertically varying anisotropic model, the azimuth of the phase velocity is the same for all layers, while the azimuths of the ray velocity are generally different. We extend the existing studies on the moveout in an azimuthally anisotropic model, accounting for the azimuthal deviation between the phase and ray velocities. We compute the lag between the azimuth of the surface offset (source-receiver vector) and the phase velocity azimuth. An effective model that replaces the multi-layer model with a single azimuthally anisotropic layer is derived, whose moveout and offset azimuth are identical to those of the layer package up to the fourth-order terms. We verify the accuracy of the approximation for small to moderate reflection angles vs. exact analytical ray tracing. The proposed approximation is in particular important when analyzing residual moveouts measured along full-azimuth common image reflection angle gathers.
Introduction

Orthorhombic models comprise subsurface anisotropy caused by vertical azimuthally-aligned fractures and layering, or by two different orthogonal sets of fractures, with or without layering. Fourth-order anisotropic moveout approximations have been studied for orthorhombic models by Tsvankin (1997), Grechka and Tsvankin (1999), Bakulin et al. (2000), Pech and Tsvankin (2004), Grechka et al. (2005). Recent advanced angle domain migrations map the seismic data into the local phase angle/azimuth domain (e.g., Koren and Ravve, 2011). Residual moveout analysis along this type of phase angle/azimuth common image gathers requires special moveout (and residual moveout) formulation. We present a new derivation for the azimuthally-dependent fourth-order effective velocity and moveout approximation in layered orthorhombic media. The moveout approximation is valid for strong anisotropy and is formulated with respect to the phase velocity azimuth.

Wave Propagation in Orthorhombic Layered Medium

Due to Snell’s law in 1D medium, the lateral slowness and azimuth of the phase velocity do not change along the whole package of layers. To derive the moveout approximation in the phase azimuth domain, we present the phase velocity as a series approximation for the greatest eigenvalue of the Christoffel matrix, and we then find the corresponding eigenvector that describes the polarization of the compressional wave. We compute the ray velocity either through the polarization vector, or through the gradient of the phase velocity magnitude in the directional space. The ray velocity, in turn, yields the lateral propagation and traveltimes through the layers. Next we expand the reflection traveltimes in series of a small surface offset, with hyperbolic and non-hyperbolic terms. The non-hyperbolic term yields azimuthally-dependent fourth-order effective velocity.

Effective Model

The phase velocity has a constant azimuth and a vertically varying dip. For a package of layers, the preferred parameter of the ray pair may be the constant (depth independent) horizontal slowness vector presented by a magnitude \( p_h \) and a phase azimuth \( \varphi \). For nearly vertical rays studied for the short offset moveout approximation, the lateral slowness magnitude is considered small. The lateral propagation through all layers can be added only geometrically, like vectors, because the individual propagations depend on the ray velocities, whose azimuths are different for various layers and all of which generally differ from the phase velocity azimuth. We distinguish two components of the lateral propagation: the lengthwise component \( h_x \) along the phase slowness azimuth, and the transverse component \( h_y \) normal to this azimuth. For each of the two components, the contributions of layers may be added. For the effective model, both components of the lateral propagation and the traveltimes coincide with those of the original multi-layer set, up to a specified order. In our model, the error terms include slowness to the fifth power for the propagation components (distances between the source and receiver on the surface) and slowness to the sixth power for the reflection traveltimes. The effective model may be described by eight independent moveout coefficients. Three of them, \( U_2, W_{2x}, W_{2y} \), belong to the second-order terms of the time series expansion, and the other five \( U_4, W_{4x}, W_{4y}, W_{4x}, W_{4y} \) are related to the fourth-order terms. The second-order parameters define the fast and slow NMO velocities and the effective azimuth of the slow velocity,

\[
V_{2,fast}^2 = \frac{U_2 + W_2}{t_v}, \quad V_{2,slow}^2 = \frac{U_2 - W_2}{t_v}, \quad \cos 2\varphi_{2,slow} = \frac{W_{2x}}{W_2}, \quad \sin 2\varphi_{2,slow} = \frac{W_{2y}}{W_2}, \quad W_2 = \sqrt{W_{2x}^2 + W_{2y}^2},
\]

where \( t_v \) is the two-way traveltime. The lengthwise component of the lateral propagation reads,

\[
h_x = U_2 p_h - \left( \cos 2\varphi W_{2x} + \sin 2\varphi W_{2y} \right) p_h + U_4 p_h^3 \]

\[
+ \left( \cos 2\varphi W_{4x} + \sin 2\varphi W_{4y} \right) p_h^3 + \left( \cos 4\varphi W_{4x} + \sin 4\varphi W_{4y} \right) p_h^5 + O(p_h^7).
\]

The corresponding transverse component reads,
The reflection traveltime $t$ for the package is,

$$h_y = (\sin 2\varphi W_{2x} - \cos 2\varphi W_{2y}) \rho_h - \frac{1}{2} \left( \sin 2\varphi W_{42x} - \cos 2\varphi W_{42y} \right) \rho_h^3$$

$$- (\sin 4\varphi W_{44x} - \cos 4\varphi W_{44y}) \rho_h^3 + O(\rho_h^5).$$

(3)

The reflection traveltime $t$ for the package is,

$$t - t_v = \frac{1}{2} U_2 \rho_h^2 - \frac{1}{2} \left( \cos 2\varphi W_{2x} + \sin 2\varphi W_{2y} \right) \rho_h^2 + \frac{3}{4} U_4 \rho_h^4$$

$$+ \frac{3}{4} \left( \cos 2\varphi W_{42x} + \sin 2\varphi W_{42y} \right) \rho_h^4 + \frac{3}{4} \left( \cos 4\varphi W_{44x} + \sin 4\varphi W_{44y} \right) \rho_h^4 + O(\rho_h^6).$$

(4)

The above global effective moveout parameters can be either obtained from azimuthally-dependent moveouts, or computed by summing the corresponding local parameters of the individual layers: $u_2, w_{2x}, w_{2y}$, and $u_4, w_{42x}, w_{42y}, w_{44x}, w_{44y}$, which depend on the thickness, azimuthal orientation and elastic properties. We have derived the explicit expressions for all eight moveout coefficients for an orthorhombic layered model, but the formulae are too lengthy for this abstract.

**Effective Fourth-Order Moveout Velocity**

The fourth-order effective velocity $V_4$ is related to the normalized coefficient $A_4$ in the moveout series and to the effective anellipticity $\eta_{eff}$. According to Tsvankin and Thomsen (1994),

$$\frac{t^2 - t_v^2}{t_v^2} = \frac{h^2}{t_v^2 V_2^2} + A_4 \frac{h^4}{t_v^2 V_2^2 V_2^2 + \beta h^2}, \quad \beta = -\frac{A_4 V_2^2}{V_h^2 - V_2^2}, \quad A_4 = -\frac{V_4^2 - V_2^2}{4 V_2^2}, \quad A_4 = -2 \eta_{eff}$$

(5)

where $V_h$ is the horizontal velocity and $V_2$ is the NMO velocity. Although this approximation was originally derived for a VTI medium, it is suitable for azimuthal anisotropy as well. Alkhalifah and Tsvankin (1995) suggested a simplified form of approximation 5 for compressional waves, where $\beta = 1 + 2 \eta_{eff}$. We present the fourth-order velocity for a layered orthorhombic structure as,

$$V_4^2(\varphi) = \frac{K_V(\varphi)}{t_v^2} \left( \frac{V_{2,slow}^4 \cos^2 (\varphi - \varphi_{2,slow}) + V_{2,fast}^4 \sin^2 (\varphi - \varphi_{2,slow})}{V_{2,slow}^4 \cos^2 (\varphi - \varphi_{2,slow}) + V_{2,fast}^4 \sin^2 (\varphi - \varphi_{2,slow})} \right)^4.$$  

(6)

Kernel $K_V$, in turn, may be presented as a bilinear form of row vector of length 5 that includes the high-order parameters, $5 \times 7$ matrix whose components depend on the low-order parameters, and column vector of length 7 that depends on the phase velocity azimuth alone.

**Numerical Tests**

We performed numerical tests to assess the accuracy of the fourth-order approximation. We compared it with the second-order approximation and with an exact ray tracing. We studied a layered structure and a single orthorhombic layer. The layer properties are presented in Tables 1 and 2, respectively.

**Table 1** Layer properties in multi-layer structure.

<table>
<thead>
<tr>
<th>#</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
<th>$\delta_3$</th>
<th>$\epsilon_1$</th>
<th>$\epsilon_2$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$V_P$</th>
<th>$\varphi_{ax}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.05</td>
<td>.05</td>
<td>0</td>
<td>.10</td>
<td>.10</td>
<td>.03</td>
<td>.03</td>
<td>2.0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>.12</td>
<td>-.08</td>
<td>.03</td>
<td>.05</td>
<td>-.03</td>
<td>.03</td>
<td>-.03</td>
<td>2.5</td>
<td>20</td>
</tr>
<tr>
<td>3</td>
<td>.04</td>
<td>-.07</td>
<td>-.01</td>
<td>.05</td>
<td>-.02</td>
<td>.02</td>
<td>-.02</td>
<td>2.75</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>.03</td>
<td>-.08</td>
<td>-.02</td>
<td>.05</td>
<td>-.04</td>
<td>.03</td>
<td>-.02</td>
<td>3.0</td>
<td>65</td>
</tr>
<tr>
<td>5</td>
<td>.06</td>
<td>-.07</td>
<td>.01</td>
<td>.05</td>
<td>-.04</td>
<td>.03</td>
<td>-.02</td>
<td>3.25</td>
<td>110</td>
</tr>
</tbody>
</table>

**Table 2** Properties of single layer.

<table>
<thead>
<tr>
<th>#</th>
<th>$\delta_1$</th>
<th>$\delta_2$</th>
<th>$\delta_3$</th>
<th>$\epsilon_1$</th>
<th>$\epsilon_2$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>$V_P$</th>
<th>$\varphi_{ax}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>.1</td>
<td>-.1</td>
<td>.05</td>
<td>.12</td>
<td>-.08</td>
<td>.08</td>
<td>-.07</td>
<td>3.0</td>
<td>110</td>
</tr>
</tbody>
</table>
For all layers, we assumed $V_P = 2V_S$, where $V_P$ and $V_S$ are vertical compressional and $x_1$-polarized shear velocities, respectively. The layer thickness is 500 m for the multi-layer model, and 1 km for the single layer. The velocities are in km/s, and the azimuths $\varphi_{ax}$ of the local orthorhombic axes $x_1$ are in degrees. The layers are numerated from top to bottom, where only the upper layer is VTI. Figures 1 and 2 present the lateral propagation and the traveltime vs. half-opening angle for the multi-layer structure. In Figure 3 we present the two-way time vs. offset. In all these cases, the azimuth of the phase velocity was zero (in the global frame), and the half opening angle was varying up to 45 degrees. In Figure 4 we present the traveltime for a fixed half-opening angle 30° and all azimuths of...
the phase velocity. In Figures 5 and 6 we plot the lengthwise and transverse components of the offset (i.e., along and across the phase velocity azimuth) for the single layer. In Figures 7 and 8 we plot the diagrams for the NMO velocity $V_2$ and the fourth-order effective velocity $V_4$ for the multi-layer structure and the single layer, respectively. Finally, in Figure 9 we present the azimuthally-dependent effective anellipticity $\eta_{eff}$.

Conclusions

We derived new relationships for moveout coefficients that constitute the fourth-order effective velocity for an azimuthally anisotropic layered model. The effective model represents a multi-layer anisotropic structure with orthorhombic layers with different azimuthal orientations by a single effective layer. It yields the same components of the lateral propagation and the same travelt ime as the original structure for any azimuth of the phase velocity, with fourth-order accuracy. It is clearly shown that even for moderate reflection angles (less than 30°) the accuracy of the second-order approximation is insufficient, and the additional fourth-order terms are essential to match the exact ray tracing solution.

References