Normalized Global Effective Parameters for Symmetric Moveout in Layered Triclinic Media

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Summary
Considering symmetric moveout where the traveltime is an even function of the offset or horizontal slowness, the local and global fourth-order normal moveout (NMO) series are governed by the normal-incidence time and eight effective parameters: three second-order and five fourth-order. Local effective parameters are related to the individual layers, while the global effective parameters account for the accumulated effect of the overburden multi layers. Local and global parameters are related by forward and inverse Dix-type transforms. We distinguish between the NMO approximations in the slowness and offset domains, where, although the formulae are different, the eight parameters are the same in both cases. To better understand the physical meaning of these parameters, we suggest a new set of intuitive normalized effective parameters, classified into two "azimuthally isotropic" and six "azimuthally anisotropic" parameters. We provide feasible ranges for the normalized parameters, thus allowing their use for controlled inversion.

Introduction
We suggest a new and intuitive set of effective parameters for azimuthally anisotropic layered media, considering seismic wave propagation with a symmetric moveout. The symmetry holds for pure-mode waves in general horizontally layered triclinic media. It also holds for converted waves in the case of layered monoclinic media and its particular cases: orthorhombic, HTI and VTI. In this case, the traveltime is an even function of the horizontal slowness or offset. In the vicinity of the normal incidence ray, traveltime, referred to as normal moveout (NMO), may be presented in the slowness domain as,

$$t\left(\psi_{\mathrm{slw}}, p_h\right)= t_{\mathrm{o}}+\frac{1}{2}V_2^2\left(\psi_{\mathrm{slw}}\right)t_{\mathrm{o}}\, p_h^2+\frac{3}{8}V_4^4\left(\psi_{\mathrm{slw}}\right)t_{\mathrm{o}}\, p_h^4+O\left(p_h^6\right), \qquad (1)$$

and in the offset domain as,

$$t\left(\psi_{\mathrm{off}}, h\right)=t_{\mathrm{o}}+\frac{h^2}{2V_2^2\left(\psi_{\mathrm{off}}\right)t_{\mathrm{o}}}-\frac{V_4^4\left(\psi_{\mathrm{off}}\right)h^4}{8V_2^8\left(\psi_{\mathrm{off}}\right)t_{\mathrm{o}}^3}+O\left(h^6\right), \qquad (2)$$

where p_h is the horizontal slowness, h is the offset, t_{o} is the two-way normal incidence time, V_2 and V_4 are azimuthally varying second- and fourth-order NMO velocities, and $\psi_{\mathrm{slw}}, \psi_{\mathrm{off}}$ are slowness and offset azimuths, respectively. In the slowness azimuth domain, the NMO velocities read (Ravve and Koren, 2017),

$$V_2^2\left(\psi_{\mathrm{slw}}\right)=\frac{U_2+W_{2x}\cos 2\psi_{\mathrm{phs}}+W_{2y}\sin 2\psi_{\mathrm{phs}}}{t_{\mathrm{o}}}, \qquad (3)$$

$$V_4^4\left(\psi_{\mathrm{slw}}\right)=\frac{2}{t_{\mathrm{o}}}\left(U_4+W_{42x}\cos 2\psi_{\mathrm{slw}}+W_{42y}\sin 2\psi_{\mathrm{slw}} +W_{44x}\cos 4\psi_{\mathrm{slw}}+W_{44y}\sin 4\psi_{\mathrm{slw}}\right), \qquad (4)$$

where U_2, W_{2x}, W_{2y} are second-order and $U_4, W_{42x}, W_{42y}, W_{44x}, W_{44y}$ are fourth-order global effective parameters. In the offset azimuth domain, relationships for the NMO velocities are more complicated, especially for the fourth-order NMO velocity (Koren and Ravve, 2017). It is convenient to split the offset h into two natural components: radial h_R along the slowness azimuth ψ_{slw}, and transverse h_T in the normal direction, as shown in the scheme of Figure 1, where $h^2=h_R^2+h_T^2$. The acute angle adjacent to the leg h_R is the azimuthal lag $\psi_{\mathrm{off}}-\psi_{\mathrm{slw}}$. We define the "global kinematic components" as the global radial and transverse offset components h_R, h_T and the traveltime t. The kinematic components are computed as functions of azimuth ψ (ψ_{slw} or ψ_{off}) and the horizontal slowness magnitude p_h, by a straightforward summation of the contributions of the individual layers. The latter are defined by the ray (group) velocities and the layer thickness. The horizontal counterpart of the ray velocity may also be split into radial and transverse components. The Cartesian components of the ray velocity depend on the derivatives of the vertical slowness with respect to two horizontal slowness components (Grechka et al., 1999), computed for normal incidence ray. For each wave type, these derivatives, in turn, are fully defined by the twenty-one stiffness components. For media and waves where the symmetry of moveout holds, the same eight global effective parameters are used in both azimuthal domains (Al-Dajani et al., 1998; Ravve and Koren, 2017, Koren and Ravve, 2017). Moreover, in the case of weak azimuthal anisotropy (WAA), the formulae in the offset domain simplify and converge to equation 3 with ψ_{off} instead of ψ_{slw}, and the azimuth ψ in equation 3 may be considered generic. The global effective parameters are related to the local (single-layer) parameters by means of forward and inverse Dix-type transforms. The local parameters govern the contribution of the given layer to the total offset and traveltime. In this study, we introduce an alternative set of global effective parameters in which the "anisotropic"

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effective parameters are normalized and classified into two groups: two "azimuthally isotropic" and six "azimuthally anisotropic" parameters. These parameters have a clearer physical interpretation of the strength of the azimuthal anisotropic effect, and they are suitable for inversion purposes, as they can be controlled and constrained.

Normalized Effective Parameters

Our proposed eight global effective parameters can be classified into two parameter subsets: azimuthally isotropic $\left\{U_2, U_4\right\}$, and azimuthally anisotropic $\left\{W_{2x}, W_{2y}, W_{42x}, W_{42y}, W_{44x}, W_{44y}\right\}$. The main advantages in using these parameter subsets are that they are generic parameters of the physical layered model, defining the second- and fourth-order NMO velocity functions in both slowness azimuth and offset azimuth domains. Furthermore, they provide simple forward and inverse Dix-type transforms. We then propose normalizing the global effective parameters in order to provide more intuitive and convenient interpretation values, which are preferable when trying to invert them from seismic data. The alternative 8 parameters are numerated with closed brackets, (1), (2), … (8).

Second-Order Effective Parameters: For the second-order effective parameters, we propose the following three alternative effective parameters,

$$\begin{aligned}&(1)\quad \bar{V}_2^2 = U_2 / t_o \,,\\ &(2)\quad E_2 = W_2 / U_2 \,, \quad W_2 = \sqrt{W_{2x}^2 + W_{2y}^2} \\ &(3)\quad \Psi_{2,H} \,, \quad \cos 2\Psi_{2,H} = \frac{W_{2x}}{W_2} \,, \quad \sin 2\Psi_{2,H} = \frac{W_{2y}}{W_2} \,,\end{aligned} \qquad (5)$$

where $\bar{V}_2$ is the azimuthally-isotropic second-order NMO velocity, E_2 is the effective elliptic parameter, and $\Psi_{2,H}$ is the effective azimuth of the high NMO velocity. Hence, the high and low NMO velocities characterizing the NMO velocities in the direction of $\Psi_{2,H}$ and in its perpendicular direction $\Psi_{2,L} = \Psi_{2,H} + \pi / 2$, respectively, are given by,

$$\begin{aligned}V_{2,H}^2 &= \left(U_2 + W_2\right) / t_o = \bar{V}_2^2 \left(1 + E_2\right) \,,\\ V_{2,L}^2 &= \left(U_2 - W_2\right) / t_o = \bar{V}_2^2 \left(1 - E_2\right) \,.\end{aligned} \qquad (6)$$

Fourth-Order Effective Parameters: For the fourth-order effective parameters, we propose the following five alternative effective parameters. The azimuthally isotropic fourth-order NMO velocity is defined by,

$$\bar{V}_4^4 = 2U_4 / t_o \,, \qquad (7)$$

and leads to the normalized azimuthally isotropic effective anellipticity,

$$(4)\quad \bar{\eta}_{\mathrm{eff}} = \frac{\bar{V}_4^4 - \bar{V}_2^4}{8\bar{V}_2^4} = \frac{2U_4 t_o - U_2^2}{8U_2^2} \,, \quad U_4 = \frac{1 + 8\bar{\eta}_{\mathrm{eff}}}{2t_o} U_2^2 \,. \qquad (8)$$

For the azimuths of the low and high NMO velocities, $\Psi_{2,H}$ and $\Psi_{2,H} + \pi / 2$, the relationships for the fourth-order NMO velocities are simplified and become domain-independent,

$$\begin{aligned}V_{4,H}^4 &= \frac{2}{t_o}\left[U_4 + \frac{W_{2x}W_{42x} + W_{2y}W_{42y}}{\sqrt{W_{2x}^2 + W_{2y}^2}} + \frac{\left(W_{2x}^2 - W_{2y}^2\right)W_{44x} + 2W_{2x}W_{2y}W_{44y}}{W_{2x}^2 + W_{2y}^2}\right] \,,\\ V_{4,L}^4 &= \frac{2}{t_o}\left[U_4 - \frac{W_{2x}W_{42x} + W_{2y}W_{42y}}{\sqrt{W_{2x}^2 + W_{2y}^2}} + \frac{\left(W_{2x}^2 - W_{2y}^2\right)W_{44x} + 2W_{2x}W_{2y}W_{44y}}{W_{2x}^2 + W_{2y}^2}\right] \,.\end{aligned} \qquad (9)$$

The corresponding azimuthally anisotropic effective anellipticities are then given by,

$$\eta_{\mathrm{eff,L}} = \frac{V_{4,\mathrm{L}}^4 - V_{2,\mathrm{L}}^4}{8V_{2,\mathrm{L}}^4} \,, \quad \eta_{\mathrm{eff,H}} = \frac{V_{4,\mathrm{H}}^4 - V_{2,\mathrm{H}}^4}{8V_{2,\mathrm{H}}^4} \,. \qquad (10)$$

Next we introduce the "high" and "low" residual effective anellipticities,

$$\begin{aligned}&(5)\quad E_{4,L} = \eta_{\mathrm{eff,L}} - \bar{\eta}_{\mathrm{eff}} \,,\\ &(6)\quad E_{4,H} = \eta_{\mathrm{eff,H}} - \bar{\eta}_{\mathrm{eff}} \,.\end{aligned} \qquad (11)$$

The last two effective parameters are the additional azimuths related to 2ψ and 4ψ,

$$\begin{aligned}&\cos 2\Psi_{42} = \frac{W_{42x}}{W_{42}} \,, \ \sin 2\Psi_{42} = \frac{W_{42y}}{W_{42}} \,, \ W_{42} = \pm\sqrt{W_{42x}^2 + W_{42y}^2} \,,\\ &\cos 4\Psi_{44} = \frac{W_{44x}}{W_{44}} \,, \ \sin 4\Psi_{44} = \frac{W_{44y}}{W_{44}} \,, \ W_{44} = \pm\sqrt{W_{44x}^2 + W_{44y}^2} \,.\end{aligned} \qquad (12)$$

The signs of W_{42} and W_{44} are chosen so that the fourth-order azimuths Ψ_{42} and Ψ_{44} are the closest to the second-order azimuth $\Psi_{2,H}$, and for a single-layer model, $\Psi_{42} = \Psi_{44} = \Psi_{2,H}$. Thus, it is convenient to choose the last two effective azimuth parameters as,

$$(7)\ \Delta\Psi_{42} = \Psi_{42} - \Psi_{2,H} \,, \ (8)\ \Delta\Psi_{44} = \Psi_{44} - \Psi_{2,H} \,. \qquad (13)$$

To obtain the inverse relationships, we introduce equations 5 and 12 into 9, and this leads to,

$$U_4 + W_{42}\cos 2\Delta\Psi_{42} + W_{44}\cos 4\Delta\Psi_{44} = V_{4,\mathrm{H}}^4\, t_\mathrm{o}/2 \;,$$
$$U_4 - W_{42}\cos 2\Delta\Psi_{42} + W_{44}\cos 4\Delta\Psi_{44} = V_{4,\mathrm{L}}^4\, t_\mathrm{o}/2 \;. \qquad (14)$$

The solution reads,

$$W_{42} = \frac{V_{4,\mathrm{H}}^4 - V_{4,\mathrm{L}}^4}{4\cos 2\Delta\Psi_{42}}\, t_\mathrm{o} \;,$$
$$W_{44} = \frac{\left(V_{4,\mathrm{L}}^4 + V_{4,\mathrm{H}}^4\right) t_\mathrm{o} - 4U_4}{4\cos 4\Delta\Psi_{44}} = \frac{V_{4,\mathrm{L}}^4 - 2\bar{V}_4^4 + V_{4,\mathrm{H}}^4}{4\cos 4\Delta\Psi_{44}}\, t_\mathrm{o} \;, \qquad (15)$$

and equation 12 is then applied to find $W_{42x}, W_{42y}, W_{44x}, W_{44y}$. Note that in the case of a single orthorhombic layer, there are no additional azimuths due to the vertical symmetry planes, and the cosines in the denominators of equation set 15 are equal to 1. For a single orthorhombic layer, equations 14 and 15 simplify to,

$$u_4 + w_{42} + w_{44} = \frac{v_{4,\mathrm{H}}^4\, \Delta t_\mathrm{o}}{2}, \quad u_4 - w_{42} + w_{44} = \frac{v_{4,\mathrm{L}}^4\, \Delta t_\mathrm{o}}{2}, \qquad (16)$$

$$w_{42} = \frac{v_{4,\mathrm{H}}^4 - v_{4,\mathrm{L}}^4}{4}\Delta t_\mathrm{o} \;, \quad w_{44} = \frac{v_{4,\mathrm{L}}^4 + v_{4,\mathrm{H}}^4}{4}\Delta t_\mathrm{o} - u_4 \;. \qquad (17)$$

where Δt_o is the local normal incidence time. We emphasize that $V_{2,H}$ and $V_{2,L}$ are the highest and lowest second-order NMO velocities, while this is not true for $V_{4,H}$ and $V_{4,L}$, which are simply the fourth-order NMO velocities in the directions $\Psi_{2,H}$ and $\Psi_{2,L}$, respectively.

Range of Effective Parameters

The validity range of the normalized effective parameters can be roughly estimated. The elliptic parameter E_2 is positive by definition, relating the high and low second-order NMO velocities to the azimuthally isotropic NMO velocity by equation 6; therefore, we assume its values are within the following range: $0 \le E_2 \le 0.5$. The effective azimuthally isotropic anellipticity may be in the range of $-0.2 \le \bar{\eta}_{\mathrm{eff}} \le 0.6$, where the negative values are less likely (due to induced anellipticity). The high and low residual anellipticities have no induced component because they are defined as differences. Therefore, negative and positive values are equally possible, and their range should be symmetric. We assume $\left|E_{4,L}, E_{4,H}\right| \le 0.4$, but in most practical cases the range will be smaller. The effective azimuth $\Psi_{2,H}$ appears in all relationships with multiplicity 2; therefore its range is $0 \le \Psi_{2,H} < \pi$. The effective residual azimuths $\Psi_{42,H}$ and $\Psi_{44,H}$ appear with multiplicities 2 and 4; in addition, we choose the signs of W_{42} and W_{44} to keep the absolute values of residual azimuths to a minimum. From this we conclude that the residual azimuths may be of any sign, and the range of their absolute values is $\left|\Delta\Psi_{42}\right| < 45^\mathrm{o}$ and $\left|\Delta\Psi_{44}\right| < 22.5^\mathrm{o}$.

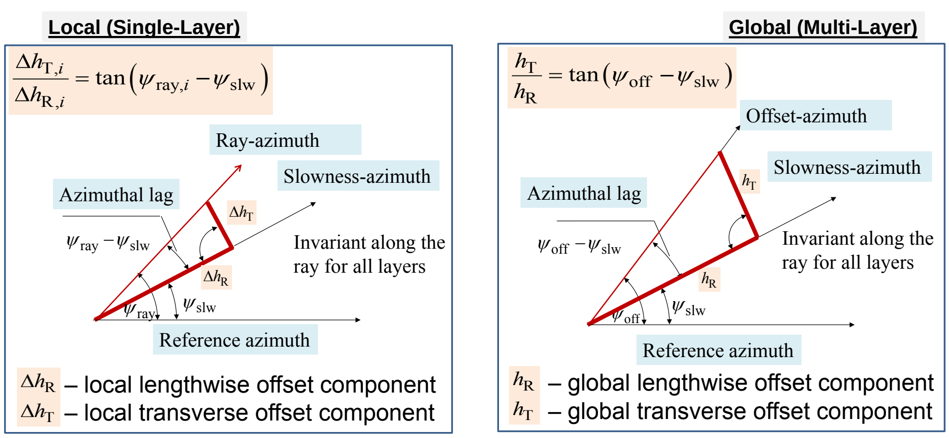

Figure 1: Scheme for local and global radial and transverse offset components with azimuthal lag between offset azimuth and slowness azimuth.

Synthetic Example

In a synthetic example, we study a ten-layer triclinic model. In order to determine the twenty-one stiffness parameters of the triclinic layers, we first define the layers as tilted orthorhombic (TOR) layers, whose properties are presented in Table 1. We then transform (rotate) the local orthorhombic stiffness matrices into the global frame to obtain the fully populated matrices (Bond, 1943). The data for each layer include the layer thickness, nine orthorhombic elastic parameters, and the three Euler orientation angles: tilt, azimuth and twist. Elastic properties include compressional velocity v_P along the tilted local axis x_3, shear parameter $f = 1 - v_{S,x_1}^2 / v_P^2$, where v_{S,x_1} is the velocity of shear waves propagating along the local x_3 and polarized along the local x_1, and seven normalized orthorhombic parameters introduced by Tsvankin (1997). The orientation angles are in degrees. In Figure 1 we plot the normalized second- and fourth-order NMO velocities and the effective anellipticity vs. slowness-azimuth and offset-azimuth. The NMO velocities are normalized with the average normal-incidence velocity.

Table 1: Elastic properties and geometry of layers in multi-layer TOR model.

#	δ_1	δ_2	δ_3	ε_1	ε_2	γ_1	γ_2	f	v_P, km/s	Δz, km	tilt	azm	tws
1	0.12	0.16	0.08	0.25	0.18	0.12	-0.10	0.74	2.0	0.35	28	36	47
2	-0.10	0.15	-0.06	0.19	0.08	0.07	-0.11	0.73	2.2	0.31	14	139	35
3	0.15	-0.13	0.09	0.29	0.12	-0.05	0.08	0.70	2.4	0.27	23	83	19
4	0.14	0.11	-0.10	0.15	-0.06	-0.06	0.07	0.72	2.6	0.19	19	74	-56
5	-0.11	0.15	0.08	-0.07	-0.16	-0.11	-0.16	0.71	2.9	0.32	15	55	-28
6	-0.08	-0.19	0.07	0.08	0.28	0.09	-0.15	0.76	3.3	0.29	18	23	31
7	-0.17	0.12	-0.04	0.24	0.16	0.10	0.12	0.78	3.6	0.36	24	-26	62
8	0.11	-0.16	-0.06	-0.20	-0.12	-0.08	-0.06	0.79	3.4	0.28	29	-96	45
9	0.19	0.14	0.05	-0.13	-0.22	0.12	-0.13	0.80	3.2	0.40	11	-84	39
10	-0.18	0.12	0.04	-0.11	0.19	0.04	0.10	0.69	3.0	0.45	30	97	79

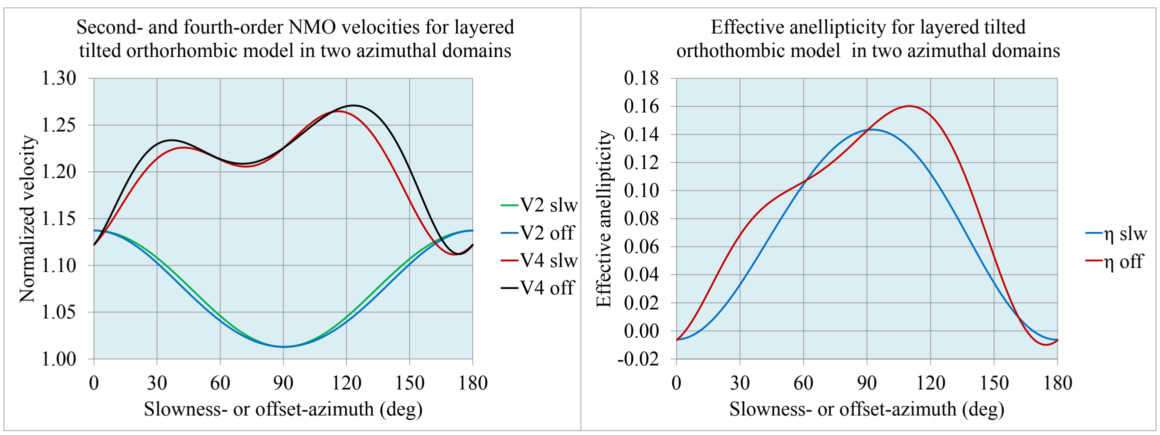

Figure 1: Second- and fourth-order NMO velocities and effective anellipticity for layered TOR model in slowness- and offset-azimuth domains.

Conclusions

The fourth-order symmetric moveout approximation for short and moderate offsets is governed by the normal-incidence time and eight effective parameters: three second-order parameters and five fourth-order. The original parameters are non-normalized and represent stacks of the corresponding local parameters of individual layers. The latter, in turn, are related to the second and fourth derivatives of the vertical slowness surfaces. They are suitable for forward and inverse Dix-type transforms. The new suggested parameterization is more intuitive and includes a single non-normalized parameter: the "azimuthally isotropic" second-order NMO velocity, which may be considered a scaling factor. The new set is more intuitive and suitable for inversion problems. Overall, the strength of the anisotropy of the effective model is governed by four effective parameters, $\bar{\eta}_{\text{eff}}$, E_2, $E_{4,H}$, $E_{4,L}$, and the non-hyperbolic traveltime is affected by all of them. The azimuthal variation of the NMO is governed by the strength of the second-order elliptic parameter E_2, the residual effective anellipticities $E_{4,\text{H}}$ and $E_{4,\text{L}}$, the effective azimuth $\Psi_{2,H}$, and two effective residual azimuths, $\Psi_{42,H}$ and $\Psi_{44,H}$. There is a unique two-way correspondence between the original and the normalized parameter sets.

EDITED REFERENCES

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