Two-Way Relationships between Slowness and Offset Azimuths in Layered Triclinic Media

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Summary

The theory for high-order, azimuthally varying, normal moveout analysis in horizontally layered general anisotropic (e.g., triclinic) media, has been recently well established for all types of pure-mode and converted waves. The moveout can be presented in both slowness-azimuth and offset-azimuth domains. In this work we present the two-way relationships between these two different azimuths, considering only the cases of symmetric moveout: triclinic layers for pure-mode waves and monoclinic layers for pure-mode and converted waves.

Introduction

Triclinic rocks are the most general anisotropic continuum, characterized by point symmetry only (at each given point, the continuum is invariant under reflection transform through the origin of the coordinate frame, located at this point) and its stiffness matrix is characterized by 21 distinct components (e.g., Slawinsky, 2015). Sedimentary layers affected by both vertical compaction and strong lateral tectonic stress are normally characterized by low symmetry (e.g., tilted orthorhombic/monoclinic or even triclinic) anisotropy. These forces can create multiple fracture sets which further lower the medium symmetry (e.g., Grechka and Kachanov, 2006, Lynn and Michelena, 2011; Jones and Davison, 2015). If fracture sets are aligned in different directions, but have their normals confined to the same plane, the medium is monoclinic (Bakulin et al., 2000); otherwise, it is triclinic (Tsvankin and Grechka, 2011). In these cases, the transverse isotropy with tilted axis of symmetry (TTI) model is insufficient to explain the azimuthal anisotropy, and, at least, a tilted orthorhombic (TOR) model representation is needed for these complex geological structures (e.g., Li et al., 2012). Note that even in cases where the individual layers are characterized by different TOR parameters, the global effective model is characterized by general anisotropy, where the symmetries of the individual layers are completely ruined. In this paper, we derive the two-way second-order series between the azimuths, $\psi_{\mathrm{off}}\left(\psi_{\mathrm{slw}}\right)$ and $\psi_{\mathrm{slw}}\left(\psi_{\mathrm{off}}\right)$. A zero-order series for relationship between the two azimuths corresponds to hyperbolic approximation for traveltime, and a second-order series for the azimuths corresponds to a fourth-order series for traveltime.

Offset and Traveltime in the Slowness Domain

The offset $\mathbf{h}$ and the horizontal slowness $\mathbf{p}_h$ are considered 2D vectors. For short and moderate offsets, the traveltime and offset may be presented as series of the horizontal slowness vector,

$$\mathbf{h}=-2\mathbf{Q}_2\cdot\mathbf{p}_h-\frac{1}{3}\mathbf{Q}_4\cdot\mathbf{p}_h^3+O\left(\mathbf{p}_h^5\right) \quad , \qquad (1)$$

$$t=t_{\mathrm{o}}-\mathbf{Q}_2\cdot\mathbf{p}_h^2-\frac{1}{4}\mathbf{Q}_4\cdot\mathbf{p}_h^4+O\left(\mathbf{p}_h^6\right) \quad , \qquad (2)$$

(Koren and Ravve, 2017), where $\mathbf{Q}_2$ and $\mathbf{Q}_4$ are second- and fourth-order tensors, representing the weighted stacks of the corresponding derivative tensors of the local vertical slowness surfaces, and dot notation stays for the full scalar product. The weight is the layer thickness Δz_i ,

$$\mathbf{Q}_2=\sum_{i=1}^{n}\mathbf{q}_{2,i}\,\Delta z_i \quad , \qquad \mathbf{Q}_4=\sum_{i=1}^{n}\mathbf{q}_{4,i}\,\Delta z_i \quad . \qquad (3)$$

The horizontal slowness and offset vectors are characterized by their magnitudes, p_h and h , and azimuths, $\mathbf{a}_{\mathrm{slw}}$ and $\mathbf{a}_{\mathrm{off}}$, and may be presented as,

$$\begin{aligned}\mathbf{p}_h&=\mathbf{a}_{\mathrm{slw}}\,p_h \ , \quad \mathbf{a}_{\mathrm{slw}}=\left\{\cos\psi_{\mathrm{slw}},\sin\psi_{\mathrm{slw}}\right\} \ , \\ \mathbf{h}&=\mathbf{a}_{\mathrm{soff}}\,h \ , \quad \mathbf{a}_{\mathrm{off}}=\left\{\cos\psi_{\mathrm{off}},\sin\psi_{\mathrm{off}}\right\} \ .\end{aligned} \qquad (4)$$

The introduction of equation 4 into 1 and 2 leads to,

$$\begin{aligned}\mathbf{h}&=-2\mathbf{Q}_2\cdot\mathbf{a}_{\mathrm{slw}}\,p_h-\frac{1}{3}\mathbf{Q}_4\cdot\mathbf{a}_{\mathrm{slw}}^3\,p_h^3+O\left(p_h^5\right) \ , \\ t&=t_{\mathrm{o}}-\mathbf{Q}_2\cdot\mathbf{a}_{\mathrm{slw}}^2\,p_h^2-\frac{1}{4}\mathbf{Q}_4\cdot\mathbf{a}_{\mathrm{slw}}^4\,p_h^4+O\left(p_h^6\right) \ .\end{aligned} \qquad (5)$$

Offset Azimuth vs. Slowness Azimuth

Multiplying the first equation of set 1 by itself and ignoring high-order terms, we obtain the offset magnitude squared,

$$h^2=4\mathbf{Q}_2^2\cdot\mathbf{p}_h^2+\frac{4}{3}\mathbf{Q}_2\cdot\mathbf{Q}_4\cdot\mathbf{p}_h^4+O\left(p_h^6\right) \quad . \qquad (6)$$

This offset magnitude may be computed as the square root of this expression,

$$h=2\sqrt{\mathbf{Q}_2^2\cdot\mathbf{a}_{\mathrm{slw}}^2}\;p_h+\frac{\mathbf{Q}_2\cdot\mathbf{Q}_4\cdot\mathbf{a}_{\mathrm{slw}}^4}{3\sqrt{\mathbf{Q}_2^2\cdot\mathbf{a}_{\mathrm{slw}}^2}}\,p_h^3+O\left(p_h^5\right) \quad . \qquad (7)$$

Note that matrix (tensor) $\mathbf{Q}_2$ is symmetric, and thus its eigenvalues are real. Eigenvalues of $\mathbf{Q}_2^2$ are squared eigenvalues of $\mathbf{Q}_2$, i.e., they are squares of real values, which means that $\mathbf{Q}_2^2$ is positive definite, and $\mathbf{Q}_2^2\cdot\mathbf{a}_{\mathrm{phs}}^2$ is a positive quadratic form for any slowness azimuth. Equation 1 may be arranged as,

$$\mathbf{a}_{\text{off}} h = -\,2\mathbf{Q}_2 \cdot \mathbf{p}_h - \frac{\mathbf{Q}_4 \cdot \mathbf{p}_h^3}{3} + O\left(\mathbf{p}_h^5\right) \quad . \tag{8}$$

or

$$\left(2\sqrt{\mathbf{Q}_2^2 \cdot \mathbf{a}_{\text{slw}}^2} + \frac{\mathbf{Q}_2 \cdot \mathbf{Q}_4 \cdot \mathbf{a}_{\text{slw}}^4}{3\sqrt{\mathbf{Q}_2^2 \cdot \mathbf{a}_{\text{slw}}^2}} p_h^2\right) \mathbf{a}_{\text{off}} = -\,2\mathbf{Q}_2 \cdot \mathbf{a}_{\text{slw}} - \frac{\mathbf{Q}_4 \cdot \mathbf{a}_{\text{slw}}^3}{3} p_h^2 + O\left(p_h^4\right) . \tag{9}$$

This leads to the offset azimuth vs. slowness azimuth,

$$\mathbf{a}_{\text{off}} = -\frac{\mathbf{Q}_2 \cdot \mathbf{a}_{\text{slw}}}{\sqrt{\mathbf{Q}_2^2 \cdot \mathbf{a}_{\text{slw}}^2}} + \left[\left(\mathbf{Q}_2 \cdot \mathbf{Q}_4 \cdot \mathbf{a}_{\text{slw}}^4\right) \mathbf{Q}_2 \cdot \mathbf{a}_{\text{slw}} - \left(\mathbf{Q}_2^2 \cdot \mathbf{a}_{\text{slw}}^2\right) \mathbf{Q}_4 \cdot \mathbf{a}_{\text{slw}}^3\right] \frac{1}{6\left(\mathbf{Q}_2^2 \cdot \mathbf{a}_{\text{slw}}^2\right)^{3/2}} p_h^2 + O\left(p_h^4\right) . \tag{10}$$

Taking into account the identity,

$$\mathbf{Q}_2 \cdot \mathbf{Q}_4 \cdot \mathbf{a}_{\text{slw}}^4 = \left(\mathbf{Q}_4 \cdot \mathbf{a}_{\text{slw}}^3\right) \cdot \left(\mathbf{Q}_2 \cdot \mathbf{a}_{\text{slw}}\right) \quad , \tag{11}$$

equation 9 may be arranged as,

$$\mathbf{a}_{\text{off}} = -\frac{\mathbf{Q}_2 \cdot \mathbf{a}_{\text{slw}}}{\sqrt{\mathbf{Q}_2^2 \cdot \mathbf{a}_{\text{slw}}^2}} + \left\{\left[\left(\mathbf{Q}_4 \cdot \mathbf{a}_{\text{slw}}^3\right) \cdot \left(\mathbf{Q}_2 \cdot \mathbf{a}_{\text{slw}}\right)\right] \mathbf{Q}_2 \cdot \mathbf{a}_{\text{slw}} - \left(\mathbf{Q}_2^2 \cdot \mathbf{a}_{\text{slw}}^2\right) \mathbf{Q}_4 \cdot \mathbf{a}_{\text{slw}}^3\right\} \frac{1}{6\left(\mathbf{Q}_2^2 \cdot \mathbf{a}_{\text{slw}}^2\right)^{3/2}} p_h^2 + O\left(p_h^4\right) \tag{12}$$

Next we take into account the general triple cross-product formula,

$$\mathbf{a} \times \left(\mathbf{b} \times \mathbf{c}\right) = \mathbf{b}\left(\mathbf{a} \cdot \mathbf{c}\right) - \mathbf{c}\left(\mathbf{a} \cdot \mathbf{b}\right) \quad , \tag{13}$$

where $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are arbitrary vectors, and in particular,

$$\mathbf{a} \times \mathbf{b} \times \mathbf{a} = \mathbf{b}\mathbf{a}^2 - \mathbf{a}\left(\mathbf{a} \cdot \mathbf{b}\right) \quad . \tag{14}$$

We rewrite equation 10 in a shorthand notation,

$$\mathbf{a}_{\text{off}} = -\frac{\mathbf{Q}_2 \cdot \mathbf{a}_{\text{slw}}}{\sqrt{\mathbf{Q}_2^2 \cdot \mathbf{a}_{\text{slw}}^2}} - \frac{\left(\mathbf{Q}_2 \cdot \mathbf{a}_{\text{slw}}\right) \times \left(\mathbf{Q}_4 \cdot \mathbf{a}_{\text{slw}}^3\right) \times \left(\mathbf{Q}_2 \cdot \mathbf{a}_{\text{slw}}\right)}{6\left(\mathbf{Q}_2^2 \cdot \mathbf{a}_{\text{slw}}^2\right)^{3/2}} p_h^2 + O\left(p_h^4\right) . \tag{15}$$

Note that vectors and tensors in equation 15 (and further equations with cross-products) are all in 2D space (in the horizontal plane), while a cross-product is defined in 3D space, so the third component of vectors in the triple products exists formally and should be set to zero. The result of the triple cross-product is in 2D space. We note also that the zero-offset component on the right side of equation 15 is normal to the coefficient of the second-order term: When the infinitesimal horizontal slowness changes, the unit vector $\mathbf{a}_{\text{off}}$ can only rotate slightly around its normal incidence value, but it preserves its unit length. The same is related to equations 17, 25 and 27. The series in equation in 6 may be inverted,

$$p_h = \frac{h}{2\sqrt{\mathbf{Q}_2^2 \cdot \mathbf{a}_{\text{slw}}^2}} - \frac{\mathbf{Q}_2 \cdot \mathbf{Q}_4 \cdot \mathbf{a}_{\text{slw}}^4}{48\left(\mathbf{D}_2^2 \cdot \mathbf{a}_{\text{slw}}^2\right)^{5/2}} h^3 + O\left(p_h^5\right) \quad . \tag{16}$$

The introduction of equation 14 into 13 leads to,

$$\mathbf{a}_{\text{off}} = -\frac{\mathbf{Q}_2 \cdot \mathbf{a}_{\text{slw}}}{\sqrt{\mathbf{Q}_2^2 \cdot \mathbf{a}_{\text{slw}}^2}} - \frac{\left(\mathbf{Q}_2 \cdot \mathbf{a}_{\text{slw}}\right) \times \left(\mathbf{Q}_4 \cdot \mathbf{a}_{\text{slw}}^3\right) \times \left(\mathbf{Q}_2 \cdot \mathbf{a}_{\text{slw}}\right)}{24\left(\mathbf{Q}_2^2 \cdot \mathbf{a}_{\text{slw}}^2\right)^{5/2}} h^2 + O\left(h^4\right) . \tag{17}$$

Equations 16 and 17 relate the offset-azimuth to the slowness-azimuth through the magnitudes of the horizontal slowness or offset, respectively. Relationships between the two azimuths have second-order accuracy, and this is exactly what is needed for the fourth-order traveltime accuracy.

Slowness Azimuth vs. Offset Azimuth

Equation 1 can be inverted, leading to horizontal slowness vs. offset.

$$\mathbf{p}_h = -\frac{1}{2}\mathbf{Q}_2^{-1} \cdot \mathbf{h} + \frac{1}{48}\mathbf{Q}_2^{-1} \cdot \mathbf{Q}_4 \cdot \left(\mathbf{Q}_2^{-1} \cdot \mathbf{h}\right)^3 + O\left(\mathbf{h}^5\right) \quad , \tag{18}$$

Multiplying it by itself and ignoring the high-order terms, we obtain the horizontal slowness magnitude squared,

$$p_h^2 = \frac{1}{4}\mathbf{Q}_2^{-2} \cdot \mathbf{h}^2 - \frac{1}{48}\left(\mathbf{Q}_2^{-1} \cdot \mathbf{h}\right) \cdot \mathbf{Q}_2^{-1} \cdot \mathbf{Q}_4 \cdot \left(\mathbf{Q}_2^{-1} \cdot \mathbf{h}\right)^3 + O\left(h^6\right). \tag{19}$$

which may be simplified to,

$$p_h^2 = \frac{1}{4}\mathbf{Q}_2^{-2} \cdot \mathbf{a}_{\text{off}}^2 h^2 - \frac{1}{48}\mathbf{Q}_2^{-1} \cdot \mathbf{Q}_4 \cdot \left(\mathbf{Q}_2^{-1} \cdot \mathbf{a}_{\text{off}}\right)^4 h^4 + O\left(h^6\right). \tag{20}$$

The horizontal slowness magnitude may be computed as the square root of this expression,

$$p_h = \frac{1}{2}\sqrt{\mathbf{Q}_2^{-2} \cdot \mathbf{a}_{\text{off}}^2}\, h - \frac{\mathbf{Q}_2^{-1} \cdot \mathbf{Q}_4 \cdot \left(\mathbf{Q}_2^{-1} \cdot \mathbf{a}_{\text{off}}\right)^4}{48\sqrt{\mathbf{Q}_2^{-2} \cdot \mathbf{a}_{\text{off}}^2}} h^3 + O\left(h^5\right) . \tag{21}$$

Equation 16 may be presented as,

$$\mathbf{a}_{\text{phs}} p_h = -\frac{1}{2}\mathbf{Q}_2^{-1} \cdot \mathbf{a}_{\text{off}} h + \frac{1}{48}\mathbf{Q}_2^{-1} \cdot \mathbf{Q}_4 \cdot \left(\mathbf{Q}_2^{-1} \cdot \mathbf{a}_{\text{off}}\right)^3 h^3 + O\left(h^5\right) . \tag{22}$$

Combining equations 21 and 22, we obtain,

$$\left[\frac{1}{2}\sqrt{\mathbf{Q}_2^{-2}\cdot\mathbf{a}_{\text{off}}^2}-\frac{1}{48}\frac{\mathbf{Q}_2^{-1}\cdot\mathbf{Q}_4\cdot\left(\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}\right)^4}{\sqrt{\mathbf{Q}_2^{-2}\cdot\mathbf{a}_{\text{off}}^2}}h^2\right]\mathbf{a}_{\text{phs}}= -\frac{1}{2}\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}+\frac{1}{48}\mathbf{Q}_2^{-1}\cdot\mathbf{Q}_4\cdot\left(\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}\right)^3h^2+O\left(h^4\right). \tag{23}$$

This leads to,

$$\mathbf{a}_{\text{phs}}=-\frac{\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}}{\sqrt{\mathbf{Q}_2^{-2}\cdot\mathbf{a}_{\text{off}}^2}}+ \left\{\left(\mathbf{Q}_2^{-2}\cdot\mathbf{a}_{\text{off}}^2\right)\left[\mathbf{Q}_2^{-1}\cdot\mathbf{Q}_4\cdot\left(\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}\right)^3\right] -\left[\mathbf{Q}_2^{-1}\cdot\mathbf{Q}_4\cdot\left(\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}\right)^4\right]\left(\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}\right)\right\} \cdot\frac{1}{24\left(\mathbf{Q}_2^{-2}\cdot\mathbf{a}_{\text{off}}^2\right)^{3/2}}h^2+O\left(h^4\right). \tag{24}$$

which may be presented with the triple cross-product,

$$\mathbf{a}_{\text{phs}}=-\frac{\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}}{\sqrt{\mathbf{Q}_2^{-2}\cdot\mathbf{a}_{\text{off}}^2}}+ \frac{\left(\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}\right)\times\left[\mathbf{Q}_2^{-1}\cdot\mathbf{Q}_4\cdot\left(\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}\right)^3\right]\times\left(\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}\right)}{24\left(\mathbf{Q}_2^{-2}\cdot\mathbf{a}_{\text{off}}^2\right)^{3/2}}\cdot h^2+O\left(h^4\right). \tag{25}$$

The series in equation 19 may be inverted,

$$h=\frac{2p_h}{\sqrt{\mathbf{Q}_2^{-2}\cdot\mathbf{a}_{\text{off}}^2}}+\frac{\mathbf{Q}_2^{-1}\cdot\mathbf{Q}_4\cdot\left(\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}\right)^4}{3\left(\mathbf{Q}_2^{-2}\cdot\mathbf{a}_{\text{off}}^2\right)^{5/2}}p_h^3+O\left(h^5\right). \tag{26}$$

The introduction of equation 26 into 25 leads to,

$$\mathbf{a}_{\text{slw}}=-\frac{\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}}{\sqrt{\mathbf{Q}_2^{-2}\cdot\mathbf{a}_{\text{off}}^2}}+ \frac{\left(\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}\right)\times\left[\mathbf{Q}_2^{-1}\cdot\mathbf{Q}_4\cdot\left(\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}\right)^3\right]\times\left(\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}\right)}{6\left(\mathbf{Q}_2^{-2}\cdot\mathbf{a}_{\text{off}}^2\right)^{5/2}}\cdot p_h^2+O\left(p_h^4\right). \tag{27}$$

Equations 15, 17, 25 and 27 constitute the second-order series for the constraints between the slowness and offset azimuths.

Traveltime in Offset Domain

When introducing equation 18 into equation 2, we obtain the traveltime in the offset domain,

$$t-t_{\text{o}}=-\frac{1}{4}\mathbf{Q}_2^{-1}\cdot\mathbf{h}^2+\frac{1}{192}\mathbf{Q}_4\cdot\left(\mathbf{Q}_2^{-1}\cdot\mathbf{h}\right)^4+O\left(\mathbf{h}^6\right). \tag{28}$$

The introduction of equation 4 into 28 leads to,

$$t-t_{\text{o}}=-\frac{1}{4}\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}^2h^2+\frac{1}{192}\mathbf{Q}_4\cdot\left(\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}\right)^4h^4+O\left(h^6\right). \tag{29}$$

Second- and Fourth-Order NMO Velocities in Slowness and Offset Domains

The traveltime may be expanded into the power series in both slowness and offset domains. In the slowness domain,

$$\frac{t-t_{\text{o}}}{t_{\text{o}}}=\frac{1}{2}V_2^2p_h^2+\frac{3}{8}V_4^4p_h^4+O\left(p_h^6\right), \tag{30}$$

while in the offset domain,

$$\frac{t-t_{\text{o}}}{t_{\text{o}}}=\frac{h^2}{2V_{2,\text{off}}^2t_{\text{o}}^2}-\frac{V_{4,\text{off}}^4}{8V_{2,\text{off}}^8t_{\text{o}}^4}h^4+O\left(h^6\right). \tag{31}$$

Taking into account equations 2, 28-30, and auxiliary set 4, we obtain the second- and fourth-order NMO velocities in the slowness domain,

$$V_2^2\left(\psi_{\text{phs}}\right)=-\frac{2}{t_{\text{o}}}\mathbf{Q}_2\cdot\mathbf{a}_{\text{phs}}^2,\quad V_4^4\left(\psi_{\text{phs}}\right)=-\frac{2}{3t_{\text{o}}}\mathbf{Q}_4\cdot\mathbf{a}_{\text{phs}}^4, \tag{32}$$

and in the offset domain,

$$V_2^2\left(\psi_{\text{off}}\right)=-\frac{2}{t_{\text{o}}}\cdot\frac{1}{\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}^2},\qquad V_4^4\left(\psi_{\text{off}}\right)=-\frac{2}{3t_{\text{o}}}\cdot\frac{\mathbf{Q}_4\cdot\left(\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}\right)^4}{\left(\mathbf{Q}_2^{-1}\cdot\mathbf{a}_{\text{off}}^2\right)^4}. \tag{33}$$

Synthetic Example

In a synthetic example, we study a ten-layer triclinic model. In order to determine the twenty-one stiffness parameters of the triclinic layers, we first define the layers as tilted orthorhombic (TOR) layers, whose properties are presented in Table 1. We then transform (rotate) the local orthorhombic stiffness matrices into the global frame to obtain the fully populated matrices (Bond, 1943). The data for each layer include the layer thickness, nine orthorhombic elastic parameters, and the three Euler orientation angles: tilt, azimuth and twist. The tilted frame is obtained from the global frame by three sequential rotations with respect to the $ZY'Z''$ axes, where the rotation angles are azimuth, tilt and twist, respectively. Elastic properties include compressional velocity v_P along the tilted (local "vertical") axis x_3, shear parameter $f=1-v_{S,x_1}^2/v_P^2$, where v_{S,x_1} is the velocity of shear waves propagating along the local x_3 and polarized along the local x_1, and seven normalized orthorhombic parameters $(\delta_1, \delta_2, \delta_3, \varepsilon_1, \varepsilon_2, \gamma_1, \gamma_2)$ introduced by Tsvankin (1997), which are extensions to Thomsen (1986) parameters of transverse isotropy. The compressional velocity is in km/s, the layer thickness is in km, and the orientation angles (tilt, azimuth and twist) are in degrees. In Figure 1 we plot the normalized second- and fourth-order NMO velocities and the effective anellipticity vs. slowness-azimuth and offset-azimuth. The NMO velocities are normalized with the average normal-incidence velocity. The average velocity, in turn, is defined as the ratio of the doubled total depth to the total two-way normal incidence traveltime.

Table 1: Elastic properties and geometry of layers in multi-layer TOR model.

#	δ_1	δ_2	δ_3	ε_1	ε_2	γ_1	γ_2	f	v_P	Δz	tilt	azm	tws
1	0.12	0.16	0.08	0.25	0.18	0.12	-0.10	0.74	2.0	0.35	28	36	47
2	-0.10	0.15	-0.06	0.19	0.08	0.07	-0.11	0.73	2.2	0.31	14	139	35
3	0.15	-0.13	0.09	0.29	0.12	-0.05	0.08	0.70	2.4	0.27	23	83	19
4	0.14	0.11	-0.10	0.15	-0.06	-0.06	0.07	0.72	2.6	0.19	19	74	-56
5	-0.11	0.15	0.08	-0.07	-0.16	-0.11	-0.16	0.71	2.9	0.32	15	55	-28
6	-0.08	-0.19	0.07	0.08	0.28	0.09	-0.15	0.76	3.3	0.29	18	23	31
7	-0.17	0.12	-0.04	0.24	0.16	0.10	0.12	0.78	3.6	0.36	24	-26	62
8	0.11	-0.16	-0.06	-0.20	-0.12	-0.08	-0.06	0.79	3.4	0.28	29	-96	45
9	0.19	0.14	0.05	-0.13	-0.22	0.12	-0.13	0.80	3.2	0.40	11	-84	39
10	-0.18	0.12	0.04	-0.11	0.19	0.04	0.10	0.69	3.0	0.45	30	97	79

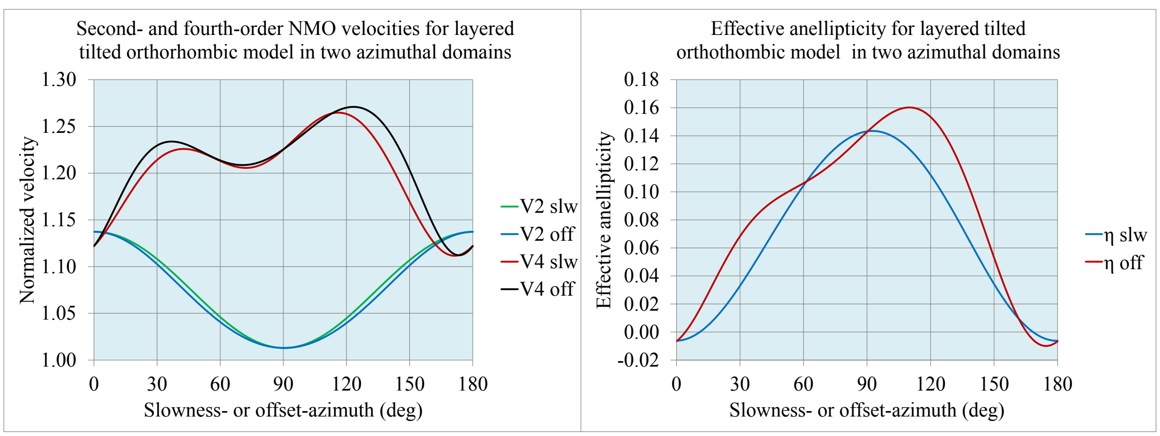

Figure 1: Second- and fourth-order NMO velocities and effective anellipticity for multi-layer TOR model in slowness azimuth and offset azimuth domains.

Conclusions

Considering symmetric moveout in a horizontally layered anisotropic media, with triclinic anisotropy for pure-mode waves and monoclinic with horizontal plane of symmetry for converted waves, we derive the coefficients of the second-order series, describing the two-way relationships between the slowness-azimuth and offset-azimuth. These second-order series correspond to the fourth-order traveltime series. The small parameter in the series may be either horizontal slowness or offset. The coefficients of the two-way azimuth series depend on the global effective parameters, which represent weighted stacks of local effective parameters, with layer thickness being the weights. The local effective parameters, in turn, depend on the second- and fourth-order derivatives of the vertical slowness surface in a single layer for a specific wave type, with respect to two lateral slowness components. The global effective parameters also define the second- and fourth-order normal moveout velocities (and effective anellipticity). The NMO velocities represent series coefficients for the offset and traveltime in the slowness-azimuth domain and for the traveltime and slowness in the offset-azimuth domain.

EDITED REFERENCES

Note: This reference list is a copyedited version of the reference list submitted by the author. Reference lists for the 2017 SEG Technical Program Expanded Abstracts have been copyedited so that references provided with the online metadata for each paper will achieve a high degree of linking to cited sources that appear on the Web.

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