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Slowness-Domain Moveout and Residual Moveout for Orthorhombic Layered Media

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Summary

Considering compressional and converted waves, we first derive approximations for the kinematical characteristics (radial and transverse offset components, traveltime and intercept time) in the slowness domain, for layered orthorhombic models. We expand the slowness surface into a series in the proximities of both the normal incidence ray and the critical (horizontal) ray, and we compute the coefficients of these series. We propose a continuous approximation function for the radial offset, valid for the entire slowness range, whose series expansions match the computed coefficients. Approximations for the other characteristics are derived from the radial offset. The residual moveout formula is obtained by keeping the intercept time identical for the background and updated (perturbed) models, and we relate the error of the vertical time (residual moveout) for a given horizontal slowness magnitude and azimuth with the residuals of the effective parameters, such as the normal moveout velocity, effective anellipticity and asymptotic properties of the updated model.

Introduction

The second- and fourth-order effective parameters of orthorhombic layered media depend on the slowness azimuth $\psi = \psi_{\mathrm{slw}}$ (Ravve and Koren, 2017a) or on the surface-offset azimuth ψ_{off} (Koren and Ravve, 2017a), and may be used to approximate short- and moderate-offset moveouts. For layered triclinic media, these parameters were recently studied in both domains by Koren and Ravve (2017b). For long offsets but for azimuthally isotropic layered media, Fomel and Stovas (2010) and Stovas and Fomel (2012) proposed generalized moveout approximations in both offset and slowness domains, respectively. These approximations were updated in the offset domain by Ravve and Koren (2017b) and in the slowness domain by Koren and Ravve (2017c) to account for nearly horizontal rays in layered anisotropic models. In this abstract, we derive the new approximations of kinematical characteristics (radial and transverse offset components, traveltime and intercept time) for a complete range of the horizontal slowness, for compressional and converted waves. The derivation of the slowness-domain residual moveout (RMO) formula is based on keeping stationary (invariant) the intercept time τ, for a given normal incidence time t_{o}, slowness-domain azimuth ψ, and horizontal slowness p_h. This yields the relation between the change of the vertical time (or depth) of a given reflector and the changes (perturbations) of the multi-effective parameters used to approximate the intercept time.

Forward Modeling Approximations

We consider horizontally-layered orthorhombic media where the layers have different azimuths of the vertical symmetry planes, but share a common horizontal symmetry plane. For a given reflector with a normal incidence time t_{o}, the kinematical characteristics (offset, traveltime and intercept time) depend on the magnitude p_h and the azimuth ψ of the horizontal slowness. The upper limit of the horizontal slowness is the critical slowness $p_c(\psi)$ of waves propagating in the horizontal symmetry plane. According to Fermat principle, nearly horizontal propagation occurs mostly in the high-velocity layer. The vertical slowness of the critical ray is zero for this layer, but finite for low-velocity layers. We compute the contributions of low-velocity layers into the offset, traveltime and intercept time. We expand the kinematical characteristics into a Tailor series, and we compute the coefficients of these series for the normal-incidence ray and for the critical ray. The series coefficients of the kinematical characteristics are based on the series for the slowness surface, $p_z(p_h, \psi)$.

Series Expansions of Kinematical Characteristics at the Ends of the Slowness Range

We define the normalized horizontal and complementary slowness $\bar{p}_h$ and $\bar{q}$, respectively,

$$\bar{p}_h = \frac{p_h}{p_c(\psi)} \, , \quad q = \sqrt{p_c^2(\psi) - p_h^2} \, , \quad \bar{q} = \frac{q}{p_c(\psi)} \quad . \tag{1}$$

For a small horizontal slowness, the series expansions of the kinematical characteristics are,

$$\begin{aligned}
h_R(\psi, p_h) &= + V_2^2(\psi) t_{\mathrm{o}} \, p_h + \frac{1}{2} V_4^4(\psi) t_{\mathrm{o},n} \, p_h^3 + O\left(p_h^5\right) , \\
h_T(\psi, p_h) &= + \frac{1}{2} \frac{dV_2^2(\psi)}{d\psi} t_{\mathrm{o}} \, p_h + \frac{1}{8} \frac{dV_4^4(\psi)}{d\psi} t_{\mathrm{o}} \, p_h^3 + O\left(p_h^5\right) , \\
t(\psi, p_h) - t_{\mathrm{o}} &= + \frac{1}{2} V_2^2(\psi) t_{\mathrm{o}} \, p_h^2 + \frac{3}{8} V_4^4(\psi) t_{\mathrm{o}} \, p_h^4 + O\left(p_h^6\right) , \\
\tau(\psi, p_h) - t_{\mathrm{o}} &= - \frac{1}{2} V_2^2(\psi) t_{\mathrm{o}} \, p_h^2 - \frac{1}{8} V_4^4(\psi) t_{\mathrm{o}} \, p_h^4 + O\left(p_h^6\right) ,
\end{aligned} \tag{2}$$

For a single layer, the intercept time is the vertical slowness scaled, $\tau = p_z \Delta z$, where Δz is the layer thickness. For nearly critical ray of a multi-layer model, the slowness surface series for the high-velocity layer is,

$$\frac{p_z^2(\bar{q}, \psi)}{p_c^2(\psi)} = M_2(\psi) \bar{q}^2 + M_4(\psi) \bar{q}^4 + O\left(\bar{q}^6\right) \quad . \tag{3}$$

We derive coefficients $M_2(\psi)$ and $M_4(\psi)$ from the Christoffel equation. For a whole multi-layer model, the series expansions for the kinematical characteristics of nearly critical ray are,

$$\frac{t}{\Delta z_H} = \frac{t_L}{\Delta z_H} + \frac{\sqrt{M_2}\, p_c}{\bar{q}} + \frac{3M_4 p_c}{2\sqrt{M_2}}\bar{q} + O\left(\bar{q}^3\right) , \qquad \frac{h_R}{\Delta z_H} = \frac{h_{R,L}}{\Delta z_H} + \frac{\sqrt{M_2}}{\bar{q}} - \frac{M_2 - 3M_4}{2\sqrt{M_2}}\bar{q} + O\left(\bar{q}^3\right),$$
$$\frac{h_T}{\Delta z_H} = \frac{h_{T,L}}{\Delta z_H} - \frac{p_c'}{p_c}\frac{\sqrt{M_2}}{\bar{q}} - \frac{M_2' p_c + \left(M_2 + 3M_4\right) p_c'}{2\sqrt{M_2}\, p_c}\bar{q} + O\left(\bar{q}^3\right), \quad \frac{\tau}{\Delta z_H} = \frac{\tau_L}{\Delta z_H} + \sqrt{M_2}\, p_c \bar{q} + O\left(\bar{q}^3\right), \tag{4}$$

where Δz_H is the thickness of the high-velocity layer, while $t_L, \tau_L, h_{R,L}$ and $h_{T,L}$ are contributions of low-velocity layers into the kinematical characteristics computed at the critical slowness $p_c(\psi)$.

Continuous Approximations for the Kinematical Characteristics

An original approximation is suggested for the radial offset, while the other characteristics are dependent,

$$h_R = \frac{A(\psi)\,\bar{p}_h + B(\psi)\,\bar{p}_h^3 + C(\psi)\,\bar{p}_h^5 + D(\psi)\,\bar{p}_h\,\bar{q} + E(\psi)\,\bar{p}_h^3\,\bar{q}}{\bar{q}} \quad , \qquad \bar{q}^2 + \bar{p}_h^2 = 1 \quad . \tag{5}$$

Coefficients A, B, C, D, E can be defined from V_2, V_4, M_2, M_4 and contributions of the low-velocity layers into the intercept time. The intercept time, traveltime and transverse offset are dependent,

$$h_R = -\frac{\partial \tau}{\partial p_h} \quad \rightarrow \quad \tau\left(p_h, \psi\right) = t_o - \int\limits_0^{p_h} \frac{\partial \tau}{\partial p_h} dp_h \quad , \quad t = \tau + h_R p_h \quad , \quad h_T = -\frac{1}{p_h}\frac{\partial \tau}{\partial \psi} \quad . \tag{6}$$

The intercept time reads,

$$\frac{\tau\left(p_h,\psi\right) - t_o}{p_c(\psi)} = -A(\psi)(1-\bar{q}) - \frac{B(\psi)(1-\bar{q})^2(2+\bar{q})}{3}$$
$$- \frac{C(\psi)(1-\bar{q})^3\left(8 + 9\bar{q} + 3\bar{q}^2\right)}{15} - \frac{D(\psi)\,\bar{p}_h^2}{2} - \frac{E(\psi)\,\bar{p}_h^4}{4} \quad . \tag{7}$$

The traveltime reads,

$$\frac{t\left(p_h,\psi\right) - t_o}{p_c(\psi)} = \frac{A(\psi)(1-\bar{q})}{\bar{q}} + \frac{B(\psi)(1-\bar{q})^2\left(3 + 4\bar{q} + 2\bar{q}^2\right)}{3\bar{q}}$$
$$+ \frac{C(\psi)(1-\bar{q})^3(3+2\bar{q})\left(5 + 9\bar{q} + 6\bar{q}^2\right)}{15\bar{q}} + \frac{D(\psi)\,\bar{p}_h^2}{2} + \frac{3E(\psi)\,\bar{p}_h^4}{4} \quad . \tag{8}$$

The transverse offset reads,

$$h_T = -\frac{t\left(p_h,\psi\right) - t_o}{\bar{p}_h p_c(\psi)}\frac{p_c'(\psi)}{p_c(\psi)} + A'(\psi)\frac{1-\bar{q}}{\bar{p}_h} + B'(\psi)\frac{(1-\bar{q})^2(2+\bar{q})}{3\bar{p}_h}$$
$$+ C'(\psi)\frac{(1-\bar{q})^3\left(8 + 9\bar{q} + 3\bar{q}^2\right)}{15\bar{p}_h} + D'(\psi)\frac{\bar{p}_h}{2} + E'(\psi)\frac{\bar{p}_h^3}{4} \quad . \tag{9}$$

Accuracy of Approximations

To estimate the accuracy of the approximations, we compute the four kinematical characteristics for compressional waves for a homogeneous orthorhombic model. The medium properties are: $\delta_1 = 0.1$, $\delta_2 = -0.08$, $\delta_3 = 0.05, \varepsilon_1 = 0.25, \varepsilon_2 = -0.15, \gamma_1 = 0.08, \gamma_2 = -0.07, f = 0.75, v_P = 3\text{km/s}, \Delta z = 1\text{km}$, where $f = 1 - v_{S1}^2 / v_P^2$, v_P is the vertical compressional velocity, and v_{S1} is the vertical velocity of shear propagating in x_3 and polarized in x_1, and Δz is the layer thickness. To estimate the accuracy, we compare the relative error between the approximations and exact numerical solutions. In Figure 1 we plot the critical slowness and azimuthally-dependent expansion coefficients of the vertical slowness for the normal-incidence and critical rays. The accuracy vs. the magnitude and azimuth of the horizontal slowness is plotted in Figure 2.

Residual Moveout

The intercept time approximation is presented in equation 7. The non-normalized horizontal slowness (along with its azimuth) and the intercept time are considered constant, while the other parameters vary: the second- and fourth-order effective NMO velocities, the parameters of the high-velocity layer: its thickness Δz_H, asymptotic characteristics M_2 and M_4, and the critical-slowness contribution of the low-velocity layers into the intercept time τ_L (for the critical slowness, the high-velocity layer does not contribute to the intercept time). However, if the measured data is not close to the critical slowness, the characteristics related to the critical ray may be assumed approximately constant and related to the background anisotropic velocity model. Hence, A, B, C, D, E in equation 7 depend only on t_o, V_2, V_4. Considering only these varying parameters, we obtain, $\tau = \tau(t_o, V_2, V_4)$. The stationarity condition for the intercept time becomes,

$$\frac{\Delta \tau}{t_o} = \frac{\partial \tau}{\partial t_o} \frac{\Delta t_o}{t_o} + \frac{\partial \tau}{\partial V_2} \frac{V_2}{t_o} \frac{\Delta V_2}{V_2} + \frac{\partial \tau}{\partial V_4} \frac{V_4}{t_o} \frac{\Delta V_4}{V_4} = 0 \quad . \tag{10}$$

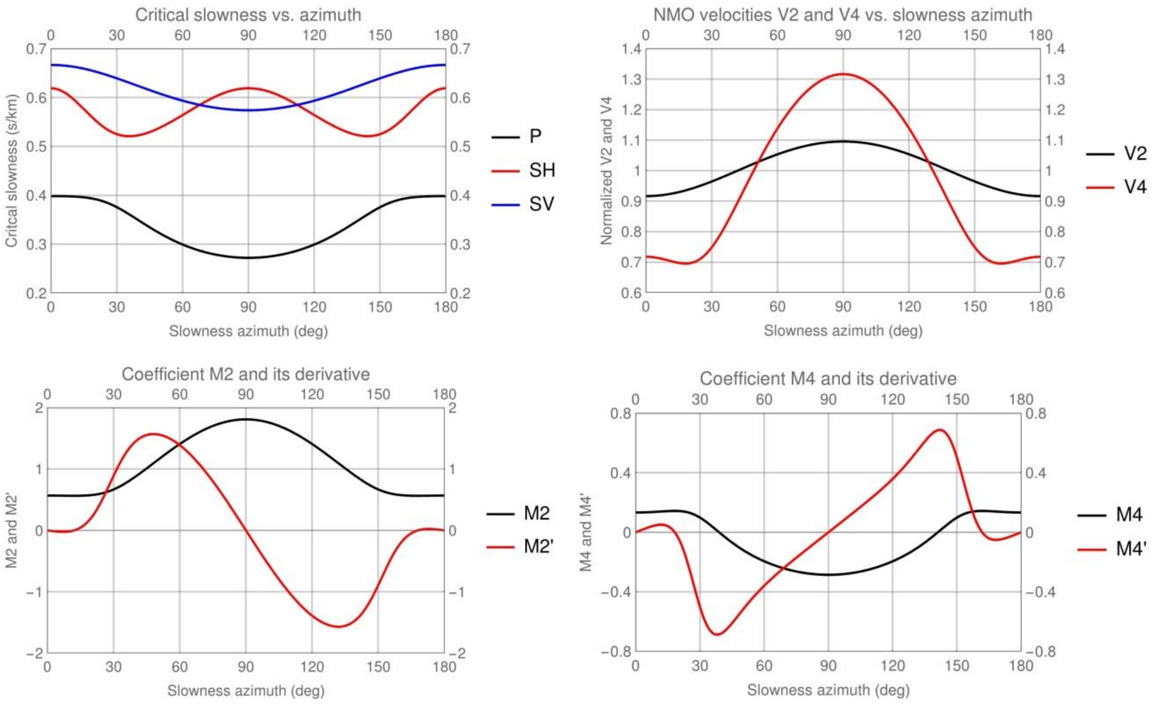

***Figure 1** Critical slowness vs. azimuth. Expansion coefficients of the vertical slowness surface for the normal-incidence and critical rays.*

Finally, the relative residual moveout formula $\Delta t_o / t_o$ measured residual vertical time becomes,

$$\frac{\Delta t_o(t_o, \psi, p_h; V_2, V_4)}{t_o} = \frac{\partial \tau / \partial V_2}{\partial \tau / \partial t_o} \frac{V_2}{t_o} \frac{\Delta V_2}{V_2} + \frac{\partial \tau / \partial V_4}{\partial \tau / \partial t_o} \frac{V_4}{t_o} \frac{\Delta V_4}{V_4} \quad . \tag{11}$$

Since we are looking for the parameter update that compensate the change in Δt_o, the signs on the right side of this equation are negated. Note that the NMO velocities, V_2 and V_4 depend on the slowness azimuth, and the three residuals, $\Delta t_o, \Delta V_2, \Delta V_4$, depend on both the magnitude and the azimuth of the horizontal slowness.

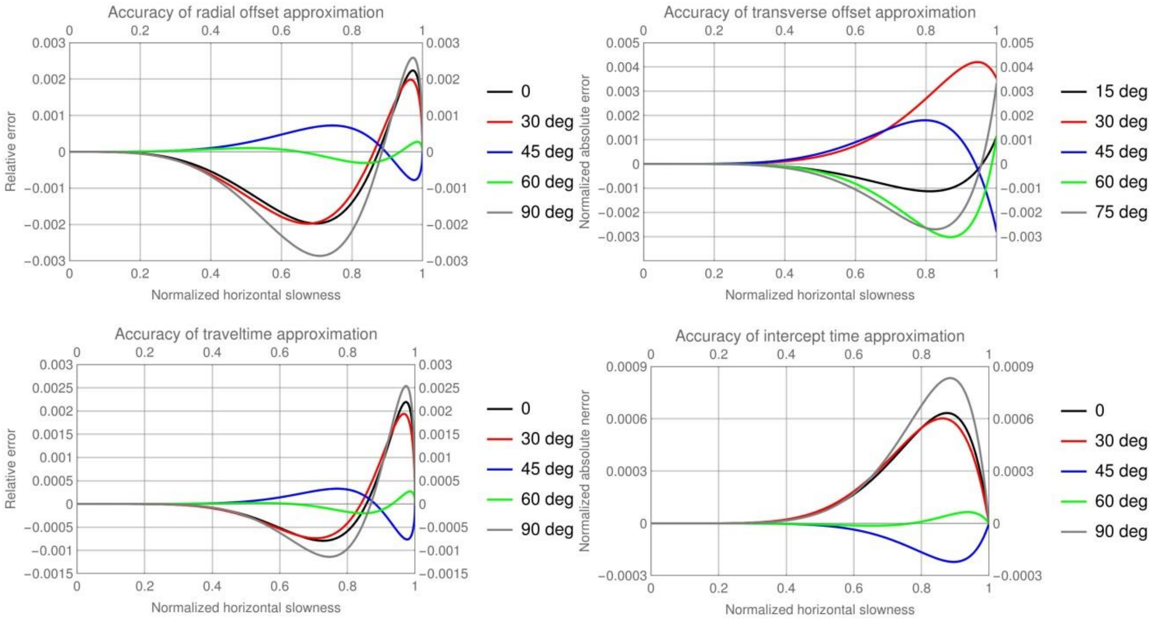

Figure 2 *Accuracy of the approximations for traveltime, intercept time and two offset components, for different slowness azimuths. Error is plotted vs. normalized horizontal slowness* $\bar{p}_h$.

Conclusions

We suggest accurate approximations for the kinematical characteristics of compressional and converted waves in layered orthorhombic models in the slowness-azimuth domain. The approximations are based on the slowness surfaces for the normal-incidence ray and the critical ray. We then derive the residual moveout formula in the slowness domain, where for all magnitudes and azimuths of the horizontal slowness, the intercept time is kept identical for the background and updated (perturbed) layered orthorhombic models. The change in the vertical time is caused by the changes of the effective parameters, primarily the second- and fourth-order azimuthally-dependent NMO velocities. All model changes depend on the magnitude and azimuth of the horizontal slowness. Updated NMO velocities allow establishing the updated interval parameters of individual layers.

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