Curved Rays Anisotropic Tomography: Local and Global Approaches
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Summary

Curved Rays Tomography updates background anisotropy velocity parameters in the time-migrated domain. The tomography uses input gather images generated by Anisotropy Curved Rays Kirchhoff Time Migration. A locally varying 1D Vertical Transverse Isotropy (VTI) model is assumed. The background anisotropy parameters are the instantaneous (interval) vertical compression velocity $V$ and the two Thomsen anisotropy parameters $\delta$ and $\epsilon$. Interval velocity (or alternatively $\delta$) is updated from short offsets reflection events, while $\epsilon$ is updated from the available long offset data. Two complementary approaches are presented in this study: local and global. In the local approach, the medium parameters are updated from top down, layer by layer, one parameter at a time. The residual anisotropy parameters, that best fit the residual moveout curves, are picked. The residual moveout includes overburden and current layer components. In the global approach, all parameters are inverted simultaneously. Due to a large number of offsets, the problem becomes over-defined, and we solve it by a constrained least-squares minimization. The cost function accounts for data and model variances, which reflect the reliability of the data and control parameter variations, respectively. The updated parameters are constrained to a feasible range.

VTI Parameters and their Range

The VTI medium is described by five Thomsen (1986) parameters, but to study the compression waves, four parameters suffice. Furthermore, the ratio between the vertical compression and shear velocity is commonly parameters suffice. Furthermore, the ratio between the vertical compression and shear velocity is commonly denoted as $\eta = \varepsilon - \delta$ for 1D medium the horizontal slowness of the ray is constant. We distinguish between initial value ray tracing (IVRT) and boundary value ray tracing (BVRT). IVRT considers a single ray with a given horizontal slowness and vertical slowness at the starting point. The goal of BVRT is to find the parameters of a specific ray pair (incident and reflected). We assume both rays emerge from the image point and arrive to the surface. The vertical time and the orientation of the reflection surface are specified at the reflection point, and the offset length and azimuth refer to the earth surface.

Initial and Boundary Value Anisotropic Ray Tracing

Ray tracing is a core element of seismic tomography. In a 1D medium the horizontal slowness of the ray is constant.

Initial Value Ray Tracing

In a 1D model, the initial value ray tracing is two-dimensional. The ray path is a curved line within a single vertical plane. Let $h$ be horizontal coordinate in this plane. The vertical coordinate is depth $z$ or vertical time $t_v$. Tracing is done numerically by solving a set of ordinary differential equations. The governing function is the Hamiltonian, which depends on two components of slowness: horizontal, $p_h = \text{const}$, and vertical, $p_v$, and on the properties of the medium, which in turn, depend only on vertical time $t_v$. The Hamiltonian function follows from the Christoffel equation for P-SV waves,

$$G(p_h, p_v, z) = -\frac{K - L - V^2 - V'^2}{2(1 - f)}$$

where parameters $K$ and $L$ are

$$K = (1 + f)[p_h^2 + p_v^2] + 2\epsilon p_h^2$$

$$L = f[p_h^2 + p_v^2] + 2\epsilon p_h^2[p_h^2 + p_v^2] - 2\delta(1 - f)p_h^2 p_v^2$$

The Hamiltonian vanishes at any point along the ray. The resolving ray tracing equations are

$$\frac{dh}{d\sigma} = \frac{2G}{G_{p_h}}, \quad \frac{dz}{d\sigma} = \frac{2G}{G_{p_v}}, \quad \frac{dp_h}{d\sigma} = -\frac{2G}{G_{p_h}}, \quad \frac{dp_v}{d\sigma} = -\frac{1}{V} \frac{\partial G}{\partial t_v}$$

where $\sigma$ is an independent integration parameter. The traveltine along the ray can be computed using

$$\frac{dt}{d\sigma} = \frac{\partial G}{\partial V} \frac{dh}{d\sigma} + \frac{\partial G}{\partial \delta} \frac{dz}{d\sigma} + p_h \frac{\partial G}{\partial p_h} + p_v \frac{\partial G}{\partial p_v}$$

Since we assume $f = \text{const}$, the vertical time derivative in equation 3 comprises three terms,

$$\frac{\partial G}{\partial t_v} = \frac{\partial G}{\partial V} \frac{dh}{d\sigma} + \frac{\partial G}{\partial \delta} \frac{dz}{d\sigma} + \frac{\partial G}{\partial \epsilon} \frac{d\epsilon}{dt_v}$$

Finally, we replace the second equation of set 3 by

\[ \frac{\partial G}{\partial t_v} = \frac{\partial G}{\partial V} \frac{dh}{d\sigma} + \frac{\partial G}{\partial \delta} \frac{dz}{d\sigma} + \frac{\partial G}{\partial \epsilon} \frac{d\epsilon}{dt_v} \]
are source and receiver locations on the earth surface, \( I \), the incident ray and azimuth \( \alpha \).

The source-receiver offset \( d_S \) be a normal to the earth surface. Note that the length (offset) and the direction (azimuth) of vector \( \overrightarrow{SR} \) are specified and not the specific locations of \( S \) and \( R \).

![Figure 1. Boundary value ray tracing](image)

In case of a tilted normal to the reflection surface, the planes of incident and reflected paths are different. The curved ray path is presented in Figure 1. Points \( S \) and \( R \) are source and receiver locations on the earth surface, \( I \) the image point, \( U \) is the projection of the image point on the earth surface, and \( N \) is the intersection of the normal line to the reflection surface (that passes through the image point \( I \)) with the earth surface. Note that the length (offset) and the direction (azimuth) of vector \( \overrightarrow{SR} \) are specified and not the specific locations of \( S \) and \( R \).

\[
\frac{dt}{d\sigma} = \frac{dz}{d\sigma} = \frac{\varepsilon G/\partial \rho}{V} \tag{6}
\]

**Boundary Value Ray Tracing**

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\[
\Delta t = \sum_{k=1}^{N} \int \left( \frac{\partial G_{pp}}{\partial V} \Delta V_k + \frac{\partial G_{pp}}{\partial E} \Delta E_k + \frac{\partial G_{pp}}{\partial \delta} \Delta \delta_k \right) d\sigma \tag{9}
\]

\[
\Delta \tau = \sum_{k=1}^{N} \int \left( \frac{\partial G_{pp}}{\partial V} \Delta V_k + \frac{\partial G_{pp}}{\partial E} \Delta E_k + \frac{\partial G_{pp}}{\partial \delta} \Delta \delta_k \right) d\sigma \tag{10}
\]

**Shift of Reflection Point in Depth**

There are two factors that cause variation of traveltine: residuals of medium properties and shift of the reflection point in depth (Koren \textit{et al.}, 1999). We assume that the zero offset traveltine is preserved. The medium properties change, and therefore the depth of the reflection point varies accordingly. Let \( \Delta z_0 \) be the one-way zero offset traveltine change caused by the medium properties variation only. It can be established by equation 11 applied for the zero offset ray. Let \( \Delta z \) be the change of depth of the reflection point. The variation of traveltine \( \Delta \tau \) caused solely by this vertical shift is

\[
\Delta \tau = \Delta \tau_0 + \Delta \tau_z = \Delta \tau_0 : \Delta \tau_z \tag{11}
\]

where \( \Delta \tau_z \) is the change of vertical "ray slowness",

\[
\Delta \tau_z = \frac{\cos \phi_S}{V_S} + \frac{\cos \phi_R}{V_R}. \tag{12}
\]
Koren, Ravve and Kosloff. Curved Ray Tomography

Conservation of the two-way zero offset traveltime reads

\[ \Delta t^{\text{ZO}} = -2 \Delta m \]

This yields an explicit expression for variation of depth,

\[ \Delta z = \frac{2}{\Delta t^{\text{ZO}}} \sum_{i=1}^{N} \int \left( \frac{\partial G}{\partial z} \Delta V_k + \frac{\partial G}{\partial \varepsilon} \Delta \varepsilon_k + \frac{\partial G}{\partial \delta} \Delta \delta_k \right) d\sigma \]

where \( \Delta t^{\text{ZO}} \) is defined in equation 12, for the zero offset ray. A similar characteristic \( \Delta t^{\text{ZO}} \) can be defined for any given nonzero offset. Variation of depth \( \Delta z \) is the same for all offsets. However, the change in traveltime, caused by this variation, is different for different offsets \( i \).

\[ \Delta t^{\text{ZO}} = 2 \frac{\Delta t^{\text{ZO}}}{\Delta t^{\text{ZO}}} \sum_{k=1}^{N} \int \left( \frac{\partial G}{\partial z} \Delta V_k + \frac{\partial G}{\partial \varepsilon} \Delta \varepsilon_k + \frac{\partial G}{\partial \delta} \Delta \delta_k \right) d\sigma \]

**Tomographic Coefficients**

Introduce the tomographic coefficients

\[ A^m_k = - \int \frac{\partial G}{\partial \sigma_m} d\sigma - \int \frac{\partial G}{\partial \sigma_m} d\sigma + 2 \frac{\Delta t^{\text{ZO}}}{\Delta t^{\text{ZO}}} \int \frac{\partial G}{\partial \sigma_m} d\sigma \]

where \( m = \{V, \delta, \varepsilon\} \) and \( A^m_k = \{A^V_k, A^\delta_k, A^\varepsilon_k\} \)

After ray tracing is done, \( A^V_k, A^\delta_k, A^\varepsilon_k \) are known values along the rays. The two-way residual traveltime reads

\[ \Delta t = \sum_{k=1}^{N} A^V_k \Delta V_k + A^\delta_k \Delta \delta_k + A^\varepsilon_k \Delta \varepsilon_k \]

Equations 16 and 17 express the linearized relation between the model parameter perturbations and residual traveltime.

**Local Approach: Single Parameter Scanning**

Local tomography is a layer stripping approach performed for single locations and for a single parameter type \( m \).

This approach is an interactive “coherency inversion” analysis type which is performed directly along the migrated image gathers (Koren et al., 1999). It is recommended to first select some sparse locations along the layer where the residual moveouts are sensitive to the model changes. Then the analysis can be performed in a batch mode for the whole layer, scanning residual model parameters within a specified range. The output is a horizon-based semblance plot for a layer, where the maximum amplitudes indicate the considered model perturbations. The resolving equations are 16 and 17, and each time only one of the residuals \( \{\Delta V_k, \Delta \delta_k, \Delta \varepsilon_k\} \) is scanned. The interval velocities (or alternatively \( \delta \)) are updated using the short-offset reflection events (\( \leq 30^\circ \)), while \( \varepsilon \) is updated using the long-offset data. Steep dips in the model contribute considerably to the sensitivity of the residual moveouts to changes in parameter \( \varepsilon \). This approach suffers from general limitations of layer stripping methods: the inaccuracies of the parameter estimation in the overburden affect the parameters of the current layer.

Figures 2 and 3 demonstrate a simple synthetic example. The vertical profile of the true VTI parameters: interval velocity, \( \delta \) and \( \varepsilon \), with the corresponding synthetic gather (calculated by anisotropy ray tracing) are shown in Figure 2. In this example, the velocity and \( \delta \) are considered known and exact, and the goal is to update \( \varepsilon \).

![Figure 2. Synthetic anisotropy model](image)

![Figure 3. Epsilon correction at the third layer: true 0.2, background 0.125, residual 0.06](image)

We set the initial guess \( \varepsilon = \delta \) at all layers. Anisotropy curved ray time migration was performed. The non-flatten gathers are shown in the right part of Figure 3. The figure shows the \( \varepsilon \) analysis in the third layer. The first and second layers have already been inverted. The corresponding \( \varepsilon \) updates are shown in the vertical and the horizon Velocity Panels. An \( \varepsilon \) histogram is performed, where the optimal residual corresponds to the maximum coherency value. The corresponding flatten event is shown in the Corrected Panel display. The residual \( \varepsilon \) values for
Global Approach

Global tomography for residual parameter update is intensively used in depth imaging (Farra and Madariaga, 1988; Stork, 1992; Kosloff et al., 1996, among others). In this section we describe a global inversion procedure for a locally varying anisotropy 1D model (time-migrated domain). The results of the local tomographic inversion are used as a background model for the global tomography. The global inversion yields all residuals simultaneously, for a fixed lateral location. The reflecting image points (elements) are stored as a set of vertical pencils. Each pencil is a vertical function, containing information about the local reflecting surfaces intersection points with the local vertical time axis. Each intersection point contains information about its vertical time value, local surface’s normal vector (dip and azimuth) and the formation index above it. In addition, at each point (node) we store the traveltime errors (residual time moveouts) related to the reflected image point. The residual times are functions of offsets or reflection angles with a given shot receiver orientation (azimuth) along the earth surface (in marine data, the azimuth is the shooting direction). Lateral location of pencils may be sparse and irregular. Vertical nodes may also be irregular and different for different pencils. Within each output interval, the residual parameters \( \Delta m \) are considered constant. The upper and lower interfaces of the intervals do not necessarily coincide with the pencil nodes. The dimensionality of the problem depends on the amount of the output intervals \( N_{\text{out}} \) and is independent on the amount of the pencil nodes \( N \). Since the problem is over-determined, the least-squares approach is used. The resolving matrix \( M \) consists of \( N_{\text{out}} \times N_{\text{out}} \) blocks, where each block has a dimension of \( 3 \times 3 \) (three parameters \( \Delta m \)). The right-side vector \( B \) consists of \( N_{\text{out}} \) blocks, each of length 3. The structure of the blocks is

\[
M_{m_k,m_l} = \sum_{n=0}^{N_{\text{out}}-1} \sum_{i=0}^{N} \frac{A_{m_k,i} \cdot A_{m_l,i}}{N^h \cdot S_{n,i,j}} + \frac{N}{3N_{\text{out}}} S_{m_k,m_l}^{m_{\text{out}},m_{\text{out}}}
\]

\[
B_{k_r}^{m_n} = \sum_{n=0}^{N_{\text{out}}-1} \sum_{i=0}^{N} \frac{A_{m_n,i} \cdot \Delta m_i}{N^h \cdot S_{n,i,j}}
\]

where \( N^h \) is the amount of offsets for pencil node \( n \), \( 0 \leq k_r \leq N_{\text{out}}-1 \) and \( 0 \leq k_s \leq N_{\text{out}}-1 \) are row index and column index, respectively, of a block in the global matrix or vector. Superscripts \( 0 \leq m_s \leq 2 \) and \( 0 \leq m_r \leq 2 \) specify the medium property. Factor \( A_{m_k,i} \) is the tomographic coefficient of medium property \( m_r \) obtained from a ray with offset index \( i \) and reflection point \( n \) within the output interval \( k_r \). Data variance \( S_{n,i,j} \) is related to reliability of traveltime residual for reflection point \( n \) and offset \( i \) (usually all offsets have the same reliability). Model variance \( S_{m_k,m_l}^{m_{\text{out}},m_{\text{out}}} \) is related to property \( m_r \) on the output interval \( k_r \), and \( \delta^2 \) is the Kronecker symbol. The standard deviation of the model parameter is assumed proportional to the interval thickness. Within the thin output interval, the information is insufficient, and thus the variation of the medium properties on this interval with respect to the background model should be limited.

Conclusions

We have described two complementary tomographic approaches for VTI parameter determination. The local tomography enables a controlled interactive estimation of the long-wavelength anisotropy parameters. In the global approach we invert simultaneously for all parameters of all output intervals using detailed residual moveout information. The reliable anisotropy parameters estimated by the local approach are used as a background (guiding) model for the global one. This makes it possible to further apply successfully the global constrained least-squares approach.

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References

EDITED REFERENCES
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REFERENCES