Combining equations 1 and 2, we obtain

$$t^2 = t_0^2 + \frac{x^2}{c^2},$$

(1)

where $x$ is the surface offset and $t_0 = 2z_0/c$ is the zero-offset traveltime.

When the CMP gathers are migrated with an incorrect velocity $v$, each offset produces a reflector image at an erroneous depth $z$ and corresponding zero-offset time $t'_0 = 2z/v$ (Figure 1), where

$$t'^2_0 = t^2 - \frac{x^2}{v^2}.$$  

(2)

Combining equations 1 and 2, we obtain

$$t^2 = t_0^2 + \frac{v^2 - c^2}{v^2c^2}x^2.$$  

(3)

Substituting $t'_0 = 2z/v$ and $x = 2z \tan \alpha$ (Figure 1), we obtain the reflection-angle move-out equation,

$$z = \frac{vt_0}{2\sqrt{1 - \left(\frac{v^2}{c^2} - 1\right)\tan^2 \alpha}}.$$  

(4)

Figure 2 presents a vertical portion of an angle gather from Kirchhoff-angle migration (Koren et al., 2002), obtained for a synthetic model with parameters $c = 2000$ m/s, $v = 2200$ m/s, and $t_0 = 1$ s. Also shown in the figure is the theoretical $z(h)$ curve according to equation 4. Distances are measured in meters and angles are in degrees.

Kinematics of local-offset gathers for horizontal reflectors with uniform velocity

Let $P(x_m, h, z, t)$ represent the downward-continued wavefield in a shot-geophone 2D prestack depth migration; $x_m$ is the midpoint coordinate, $h$ is half of the local offset, and $t$ is time. For a given CRP location $x_m$, the local-offset gather $g(h, z)$ is obtained according to

$$g(h, z) = P(x_m, h, z, t = 0).$$  

(5)

Figure 3 shows the geometry of a reflection event on a local-offset gather for a horizontal reflector in a uniform medium. One can verify from the figure and equation 1 that

$$z = v \sqrt{\frac{t^2}{4} - \left(\frac{x}{2} - h\right)^2} = v \sqrt{\frac{t_0^2}{4} + \frac{x^2}{4c^2} - \left(\frac{x}{2} - h\right)^2}.$$  

(6)

An image is formed when the depth $z$ is stationary with respect to change in offset. Setting $dz/dx$ to zero in equation 6 yields a relationship between the surface offset $x$ and half-offset $h$:

$$x = -\frac{2h c^2}{v^2 - c^2}.$$  

(7)
Substituting equation 7 into equation 6 yields
\[ z = v \sqrt{\frac{t_0^2}{4} - \frac{h^2}{v^2 - c^2}}. \] (8)

This equation relates the depth of a reflection event on a local-offset gather to the local offset.

Figure 4 shows a vertical portion of a local-offset gather obtained from shot-geophone wave equation prestack depth migration with the same model parameters as in the previous section. Also shown in Figure 4 is the predicted \( z(h) \) curve according to equation 8. As the figure demonstrates, the match is very good.

**Kinematics of reflection-angle gathers**

In the procedure outlined by Sava and Fomel (2003), angle gathers are created from the local-offset gathers \( g(h, z) \) by a
Figure 5. Geometrical configuration for the production of angle gathers from local-offset gathers.

slant-stack operation. This can be written

$$\tilde{g}(p, z') = \sum_h g(h, z' - ph).$$  \hfill (9)

They claim that $p = \tan \alpha$, where $\alpha$ is the reflection angle.

Figure 5 shows the geometry for producing the slant-stack mapping. An event $(z, h)$ on a local-offset gather is mapped to slope and depth $(z', p)$ according to

$$z' = z - hp.$$ \hfill (10)

For the horizontal reflector model and a given value of $h$, $z$ is obtained from equation 8 and $p$ is obtained by taking the derivative of this equation with respect to $h$. This allows us to predict the move out curve $z'[\arctan(p)]$ on a migrated angle gather.

Figure 6 shows a vertical portion of a depth-migrated angle gather for the previous model parameters. Also shown in the figure are the predicted $z'[\arctan(p)]$ curve according to the procedure of Sava and Fomel as well as the $z(\alpha)$ curve according to equation 4. As Figure 6 shows, these two curves deviate quite noticeably. The depths of the migrated gather match the depths according to equation 10 and not the correct depth curve according to equation 4.

Therefore, the procedure of Sava and Fomel does not produce correct reflection-angle gathers in the presence of velocity errors. However, these gathers may give a good indication of the direction of the velocity errors (e.g., events curve upward for too low a velocity and curve downward for too high a velocity).

Sava and Fomel suggest a second procedure to generate slowness gathers $p_b = k_b/\omega$ during migration. Although these gathers do not give the reflection angle directly, we recommend using them with velocity-determination methods that require precise moveout, such as depth tomography.

ACKNOWLEDGMENTS

The authors are grateful to Zvi Koren for numerous helpful discussions.

REFERENCES


Reply to the discussion

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We thank Bartana et al. for their interest in our paper and for suggesting an interesting topic for discussion.

In our paper (Sava and Fomel, 2003), we do not address the issue of using wave-equation angle gathers for velocity

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